Short Circuit Calculations in Power System

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SHORT CIRCUIT (FAULT) CONCEPTS

Short circuits or faules in power system may be due to the following reasons:

1. Insulation failures due to:
   (a) lightning
   (b) Switching surges
   (c) pollution due to contamination of dust, salt spray or humidity rise.
   (d) equipment failures.
2. Mechanical causes:
   (a) Trees falling on the electric wires
   (b) Vehicular collision with the poles or towers.
   (c) Conducting materials comes in contact with a bare conductor due to human throw or cranes
3. Meteorological causes:
   (a) lightning stroke
   (b) Thunder storms
   (c) Heavy wind blow (conductor swings)

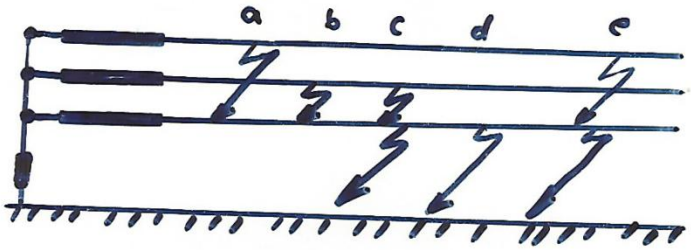
Fault classifications

1) permanent fault : fault causes complete damage in insulations and equipment of the network and it must be changed with new one (repair time is large).

2) Temporary fault (instant fault) : fault still for very short time and then cleared off.
   (a) Recursive fault : occures more than one time
   (b) Nonrecursive fault : occures only one time

Also the fault can be classified according to occurance probability :

(a) Three-phase (symmetrical) fault.
(b) Line to Line fault
(c) Double-line to ground fault
(d) Single-line to ground fault
(e) Three-phase to ground fault.



Fault calculations Assumptions & Approximations

1. Resistances of the generators, transformers, short cables, parallel over-head lines are ignored.

   However when fault-current path include considerable resistance (fairly long cables or long overhead lines or such with small conductor cross section or small transformers), these should not be neglected and the resistance must be taken into considerations.

   - impedances of busbars in switchgears for service voltage over 1KV can almost always be neglected as they are usually small compared with the much higher values for the generators, transformers and reactors.

2. Shunt elements in the transformer model are neglected; that is, magnetizing currents and core losses are neglected.
3. Shunt capacitances in the transmission line model are neglected.
4. Transformers are set at nominal tap positions
5. All internal voltage sources are set equal to $1.0 \angle 0^\circ$.
6. All prefault load currents are neglected.
7. All non-rotating impedance load are neglected.
8. Induction motors are either neglected (especially for motor rated 50 hp or less) or represented in the same manner as synchronous machines.

The generator Reactances

The variation of short circuit current with time in a generator, three different generator reactance are necessary:

the subtransient reactance X_d''
the transient reactance X_d'
the synchronous " X_s

X_d'' comprises the leakage reactance of the stator and rotor windings of the generator + the damper winding and solid parts of the rotor body. $X_d'' \simeq 0.12$ for turbo-generators

x'_d comprises the leakage reactance of the stator and excitation winding of the generator. $x'_d > x''_d$ always. (0.18 for turbo-generator).

X_d, X_s is the total reactance of the armature winding (stator leakage reactance + armature reaction reactance) of the generator. Its value for turbo-generator $\approx$ 1.C .

N.B. All these reactances come under the concept of positive-sequence reactances as will be explain later. Therefore, they are applicable to generators only, and are useful to determine the short circuit current for a specific time instant.

The index "d" means that the reactances refer to a position of the rotor such that the axis of the rotor winding coincides with the axis of the stator winding. This is called the direct axis.

In addition to these reactances the generator also has reactance in the corresponding quadrature axis which, however, have no practical importance for short circuit calculations.

Short-circuit current characteristics

The short-circuit current in electrical network is similar to the current transient in RL network (circuit) shown in Fig.1 when switch S is closed

$$V_m \sin(\omega t + \alpha) = Ri + L\frac{di}{dt}$$

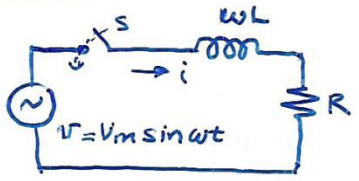


Fig.1

The solution for this equation is

$$i = \frac{V_m}{|Z|}\left[\sin(\omega t + \alpha - \phi) - e^{-\frac{Rt}{L}} \sin(\alpha - \phi)\right]$$

where $Z = \sqrt{R^2 + (\omega L)^2}$, $\phi = \tan^{-1}\frac{\omega L}{R}$.

The worest case occurs when $\alpha = \pm\frac{\pi}{2}$, here the voltage is it maximum positive value . Fig. 2 shows the variation of short-circuit current with time

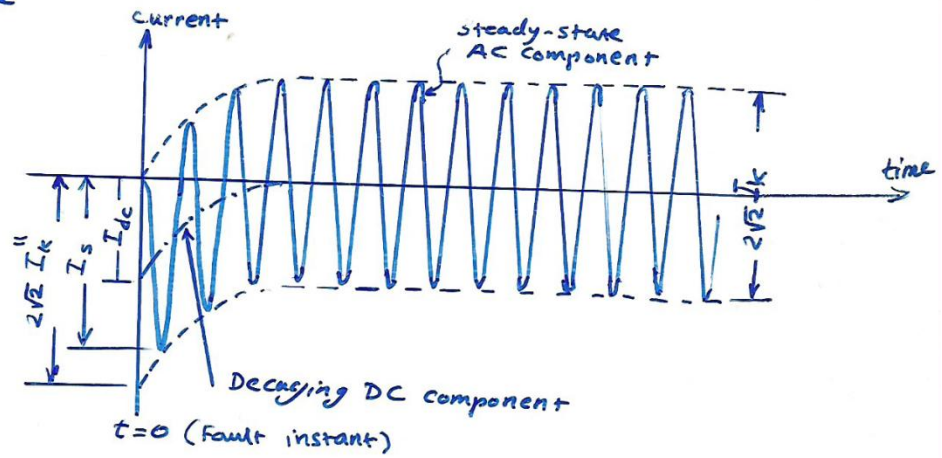


Fig. 2

where :

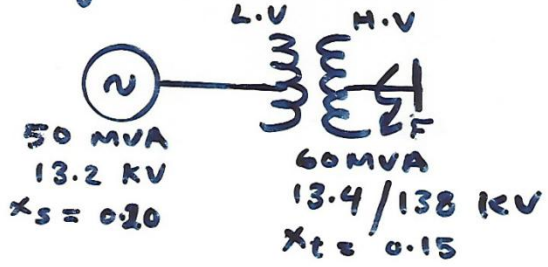
I_k'' = Initial symetrical short-circuit current (which is the r.m.s value of the short-circuit current at the instant of S.C).

I_k = steady-state short-circuit current (r.m.s).

I_s = peak short-circuit current (is the highest inst. value of the s.c current after S.C occurs).

I_{dc} = Initial value of DC component $= \sqrt{2}\, I_k$

Example: A 3-ϕ synch. generator is rated 50MVA, 13.2kV, 0.8 p.f lagging and has a synchronous reactance of 20%. The generator is connected to a 60 MVA, 13.4/138-KV 3ϕ-transformer having a reactance of 15%. The generator is initially operating at no load and rated terminal voltage. Determine the short circuit current supplied by the generator and its terminal voltage.



Solution: Choose the generator ratings as a base quantities, thus $MVA_{base} = 50$ and $V_{base} = 13.2$ KV (line-to-line).

$$\therefore \text{Base current } I_{base} = \frac{MVA_{base}}{\sqrt{3}\, V_{base}} \times 10^3$$

$$= \frac{50 \times 10^3}{\sqrt{3} \times 13.2} = 2187 A.$$

The terminal voltage of the generator is taken as reference phasor, thus

$$V_t = 1.0 \angle 0^\circ \text{ p.u.}$$

Because the generator is initially operating at no load, the internal voltage is equal to the terminal voltage.

$$\therefore E_a = V_t = 1.0 \angle 0^\circ$$

When a 3-ϕ fault occurs at the H.V side of the transformer, the internal voltage of the generator is assumed to remain constant in

determining the short circuit current. The transformer reactance is converted to per unit referred to the chosen bases as follows:

$$X_{t_{new}} = (X_t)_{old} \left(\frac{MVA_{base_{new}}}{MVA_{base_{old}}}\right)\left(\frac{KV_{old}}{KV_{new}}\right)^2$$

$$= (0.15)\left(\frac{50}{60}\right)\left(\frac{13.4}{13.2}\right)^2 = 0.129 \text{ P.U.}$$

Therefore the total series reactance X_T is given by

$$X_T = X_S + X_t = 0.20 + 0.129 = 0.329 \text{ p.u.}$$

$$\therefore I_{sc} = \frac{E_a}{jX_T} = \frac{1.0\angle 0^\circ}{j\,0.329} = 3.04\angle -90^\circ$$

$$= 3.04 \times 2187 = 6648 \text{ A.}$$

The terminal voltage of the generator is calculated as

$$V_t = jX_t I_{sc} = j(0.129)(3.04)\angle 90^\circ$$

$$= 0.39\angle 0^\circ \text{ p.u}$$

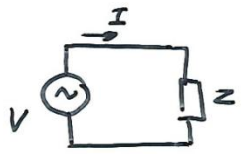
$$= 0.39 \times 13.2 = 5.15 \text{ KV line-to-line}$$

THE PER-UNIT SYSTEM

The p.u. value of any quantity is defined as the ratio of that quantity to an arbitrarily chosen value having the same dimensions and called the base value.

(1) Single-Phase System

If a single-phase supply of voltage V feeds a current I amperes to a load impedance $Z\ \Omega$



If V_b volts is chosen, then the p.u. value of the supply voltage is

$$V_{p.u} = \frac{V}{V_b}.$$

Similarly $I_{p.u} = \frac{I}{I_b}$ (Always we choose two quantities V_b & I_b)

$$\therefore Z_b = \frac{V_b}{I_b} \quad \& \quad Z_{p.u} = \frac{Z}{Z_b} = \frac{V/I}{V_b/I_b} = \frac{V/V_b}{I/I_b} = \frac{V_{p.u}}{I_{p.u}}.$$

Thus the per-unit system obeys ohm's law (and hence all the other networks laws).

Base Volt-Amp $S_b = V_b I_b$

$$\& \quad S_{p.u} = \frac{S}{S_b} = \frac{VI}{V_b I_b} = V_{p.u} \cdot I_{p.u}$$

(2) Three-phase System:

The per-unit system of units can also be applied to a 3-phase circuit.

For balanced 3-phase circuits, the equivalent per-phase "STAR" (line-to-neutral) values are used for all original and base quantities.

N.B When dealing with balanced 3-phase circuits, it is always be assumed that the circuit is STAR-CONNECTED. i.e the impedances take line current at line-to-neutral voltage.

For a 3-phase balanced system:

* P.U. value of the line-to-line voltage = p.u. value of the line-to-neutral voltage.

* P.U. value of the 3-phase volt-ampers = p.u. value of the volt-ampers/phase.

provided that always the base values form a balanced 3-phase system. i.e:

$$V_{L\,base} = \sqrt{3}\ V_{ph\,base}$$

$$MVA_{base}\Big|_{3\text{-phase}} = 3\ MVA_{base}\Big|_{per\ phase}$$

In the p.u. system, the MVA (whether 3-phase or per phase) equal the product of the voltage (whether line or phase) and the current and there are no factors such as 3 or $\sqrt{3}$.

$$MVA_{p.u} = \frac{MVA}{MVA_{base}} = \frac{\sqrt{3}\, I_L V_L}{\sqrt{3}\, I_{base} V_{L\,base}} = \frac{I_L}{I_{base}} \times \frac{V_L}{V_{L\,base}}$$

$$= I_{p.u} \cdot V_{p.u}$$

$$\therefore MVA = I_{p.u} \cdot V_{p.u}\ MVA_{base}.$$

Note that only two (any two) base quantities are chosen, and all other base quantities can be derived from the two specified.

In 3-phase system 3-phase MVA and line KV are the two quantities usually specified (as these are usually given on the rating plate of any machine), but but the base quantities chosen are the corresponding phase values, or 3-phase MVA and phase voltage. Thus if the base values are taken as 100MVA (3-phase) at 132 KV (line) then:

1 p.u. voltage = $132/\sqrt{3} = 76.21$ KV/phase

1 p.u. current = $100/(\sqrt{3} \times 132) = 0.4373$ KA.

1 p.u impedance = $76.21/0.4373 = 174.3\ \Omega$/phase (star)

Limiting of short-circuit currents in Low Voltage System

The effects of short circuits in a system depend on the magnitude and duration of the short-circuit current. For high load intensity and hence correspondingly high power concentration with distribution transformers working in parallel, attention should be paid to the level of short-circuit available on the l.v side such that this does not exceed 50 KA (RMS) [For distribution systems in Iraq], as most factory-made switchboards are designed for this.

Shor-circuit limiting measures:

Design measures which can be incorporated to limit the short-circuit current are:

1. Segregation of the transformers to form part systems which can be coupled or interconnected in the event of part of the system being inoperative due to fault; this can be done by:
   (a) Introlocking arrangements to ensure that interconnection do not produce a fault capacity condition which exceeds 50 KA (RMS)
   (b) selection of higher impedance transformers.
2. Using protective devices:
   To disconnect short-circuit currents and to limit stress on switchboards and apparatus, the following devices are normally used:
   (a) Fuses
   (b) Circuit breakers with thermal and electro-magnetic overcurrent releases.
   (c) Switch combinations, consisting of circuit breaker and back-up fuses; air-break contactors, bimetallic releases and back-up fuses

Symmetrical 3-phase Short Circuit

This type of fault is defined as a simultaneous short-circuit across all three-phases.

Occurance probability: Infrequent.

Importance: it is an important type because:
(1) it results in an easy calculation
(2) gives pessimistic answer to the worest case with regard to **FAULT LEVEL**

FAULT LEVEL CALCULATIONS

This is always be done by symmetrical 3-ph short-circuit calculation.

Consider the system shown in Figure below:

V_r is the rated voltage
E is the voltage behined a reactance x/phase.

Fault current/phase $I_f = \frac{E}{X}$ amp.

At no load conditions and if the pre-fault current is neglected (assumptions). then

$E = V_r$.

$\therefore$ total fault level $= 3 . V_r I_f = 3 . V_r \frac{E}{X}$ Volt-amp

changing to p.u. notation:

$$I_{f_{p.u}} = \frac{I_f}{I_{Base}}, \quad E_{p.u} = \frac{E}{V_{Base}} \quad \& \quad X_{p.u} = \frac{X}{X_{Base}}$$

and $I_f = \frac{E_{p.u}}{}$

Ifpu =Epu/Xpu

Base Volt-ampere/phase $= V_{Base}\, I_{Base}$

If the base voltage = rated voltage ($V_{Base} = V_r$)

$$\therefore (VA)_{f_{p.u}} = \frac{V_r \cdot \frac{E}{X}}{V_{Base}\, I_{Base}} = \frac{E}{V_{Base}} \cdot \frac{V_r}{I_{Base} \cdot X}$$

$$= \frac{E}{X} \cdot \frac{1}{I_{Base}} = \frac{I_f}{I_{Base}} = I_{f_{p.u}} = \frac{E_{p.u}}{X_{p.u}}.$$

Since $E : V_r = 1.0 \angle 0^\circ$

Hence $I_{f_{p.u}} = \frac{1}{X_{p.u}}$

& $(VA)_{f_{p.u}} = \frac{1}{X_{p.u}}$.

Example: In Figure below, four identical alternators are connected in parallel to the same busbar. Each rated at 11 KV, 25 MVA, and each having a (subtransient) reactance of 16% on it continuous thermal rating. Find the 3-ph fault level at one of the outgoing feeders.

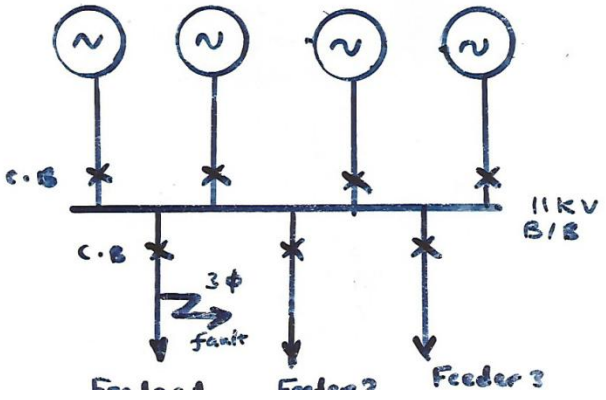


Fig.
Three phase fault a-b-c
Actual system single line diagram.

Solution :

Let Base MVA = 25

Base Voltage = 11 KV.

The fault MVA from one alternator is

$$(VA)_{f\,p.u} = \frac{1}{x_{p.u}} = \frac{1}{0.16} = 6.25 \text{ p.u.}$$

$$\therefore (VA)_f = (VA)_{p.u} \times (VA)_{base}$$

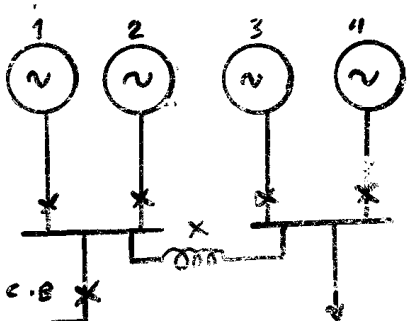
$$= 6.25 \times 25 = 156 \text{ MVA}.$$

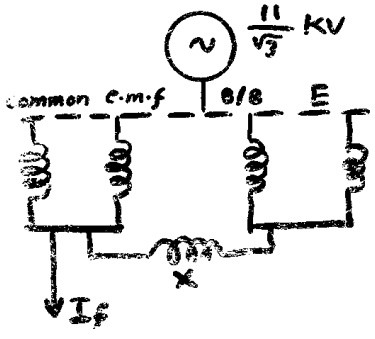
$\therefore$ Total fault MVA = $156 \times 4 = 625$ MVA.

CURRENT LIMITING REACTORS

Current limiting reactors are used to reduce the fault level at a given section in a power system.

In the above example, the fault level is 625 MVA, whereas the nearest circuit-breaker standard ratings are 500 MVA & 750 MVA. If for economic reasons, it is necessary to use 500 MVA circuit breaker. Then the fault current must be reduced by increasing the system reactance. This can be done by adding seperate reactances either (1) in series with each alternator (2) in series with each feeder or (3) to sectionalize the busbar and connect sections via a busbar reactor as shown in Figure below :





Fault abc load

Example: Using the data of the previous example but with reactor as shown in the above figure. Calculate the reactance of the current limiting reactance X to limit the fault level on a feeder to 500 MVA.

Solution:

Base 11 KV, 25 MVA.

Two alternators 1&2 will supply $\frac{625}{2} = 312.5$ MVA directly to the fault.

alternators 3 & 4 section must be limited to

$500 - 312.5 = 187.5$ MVA.

or in P.u $= \frac{187.5}{25} = \frac{1}{X_t}$

where X_t is the total reactance of the right hand section. Thus $X_t = \frac{25}{187.5} = 0.133$ p.u.

or $X_t = 13.3\%$.

Since alternators 3&4 are in parallel, they have a combined reactance of $\frac{0.16}{2} = 0.08$ p.u thus the busbar reactor must provide the remaining reactance i.e $0.133 - 0.08 = 0.053$ p.u

The ohmic value of the reactor can be calculated as follows:

base voltage $= 1$ p.u $= \frac{11}{\sqrt{3}} = 6.36$ KV.

base current $= 1$ p.u $= \frac{25}{\sqrt{3} \times 11} = 1.31$ KA.

base reactance $= 1$ p.u $= \frac{6.36}{1.31} = 4.86\ \Omega$.

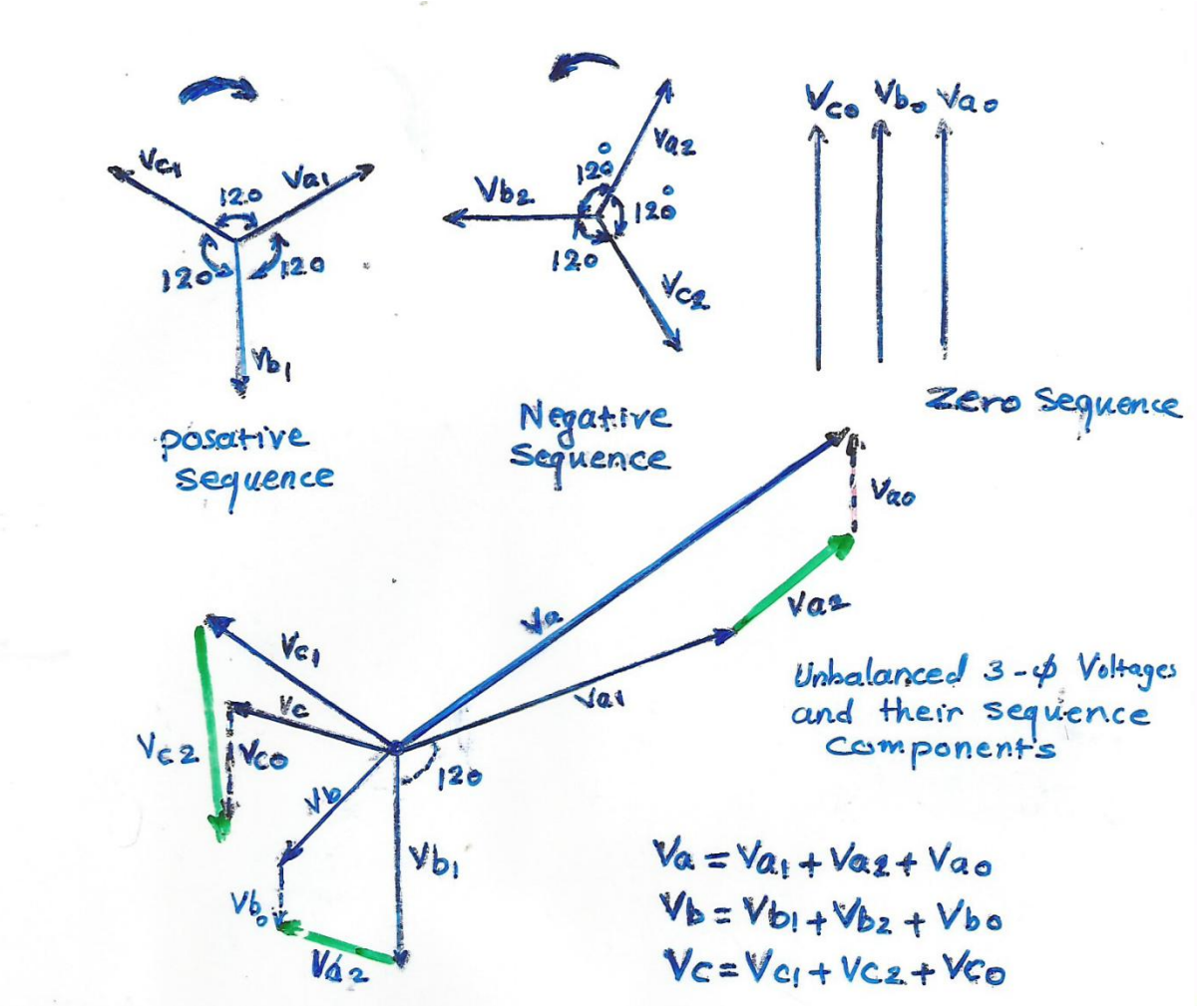
and 0.053 p.u ractance $= 4.86 \times 0.053 = 0.25\ \Omega$.

N.B. Current-limiting reactors should have a reactance high enough to limit the fault current, but not so high as to cause excessive voltage drop due to load current.

current

Unbalanced faults

Symmetrical components



Taking phase (a) as ref.

let $\lambda = 1\angle 120^\circ$, $\lambda^2 = 1\angle 240^\circ$

$V_{b1} = \lambda^2 V_{a1}$

$V_{c1} = \lambda V_{a1}$

$V_{b2} = \lambda V_{a2}$

$V_{c2} = \lambda^2 V_{a2}$

$V_{b0} = V_{c0} = V_{a0}$

$\therefore$ $V_a = V_{a1} + V_{a2} + V_{a0}$

$V_b = \lambda^2 V_{a1} + \lambda V_{a2} + V_{a0}$

$V_c = \lambda V_{a1} + \lambda^2 V_{a2} + V_{a0}$

In Matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

positive - Sequence, Negative - Sequence and zero - Sequence Impedances

1. The positive - sequence Impedance Z_1: is the impedance of the items of plant equipment as measured under balanced load conditions.

For Lines Z_{1L} = per - phase impedance

For Transformers and Reactors $Z_{1T} \ni Z_{1R}$ = Short circuit impedance

For Generators $Z_1 = x_d''$ or x_d' or x_d depending on the instant of fault occurance.

2. The negative - Sequence Impedance Z_2:

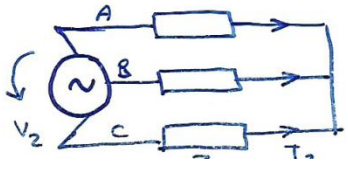
For Lines, Transformers & Reactors $Z_2 = Z_1 = R_1 + jX_1$

For generators, Z_2 is the impedance the machine has, if a voltage system with reverse sequence is applied to it when running in synchronism. This reverse - rotating system has double the relative speed with respect to the rotor, whereas the relative speed of the positive sequence system rotating in the same direction as the rotor is zero.

For Turbogenerators $Z_2 \simeq Z_1$ at the instant of incidance of the fault

For Salient - pole generators with damper windings $Z_2 = 1.2 Z_1 = (1.2 x_d'')$

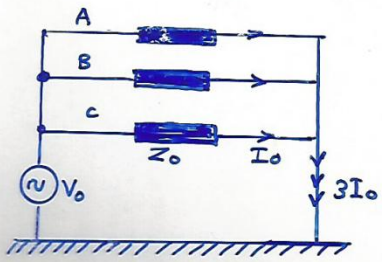
For Salient - pole generators without damper windings $Z_2 = 1.5 Z_1$



Z_2 I_2

3. The Zero - Sequence impedance

The zero sequence impedance Z_0 can be determined either by measurement or by calculation. The 3-ϕ network being connected in parallel and a single-phase alternating voltage applied to them.



Zero-Sequence System

(i) For Generators $Z_0 < X_d''$ $\left[Z_0 = (\frac{1}{3} \sim \frac{1}{6}) X_d''\right]$.

(ii) For Transformers Z_0 depends on the type of connection:

$Y-\Delta$ $\quad Z_0 = 0.8 - 1$ times Z_{1tr}

$Y-$Zig-Zag $\quad Z_0 = 0.1\, Z_{1tr}$.

$Y-Y$ with tertiary winding designed for $\frac{1}{3}$ of transf. rating. $\quad Z_0 = 2.4\, Z_{1tr}$.

Three-limb $Y-Y$ without tertiary winding $\quad Z_0 = (5-10) Z_{1tr}$

N.B: This type of transformers is not recommended for system earthing because a large leakage flux flows through the transformer tank walls and consequent heating.

For Shell-type transformers & Bank (3 single-phase) transformers } $Y-Y$

$Z_0 \simeq$ Open circuit impedance. { Also unsuitable for earthing

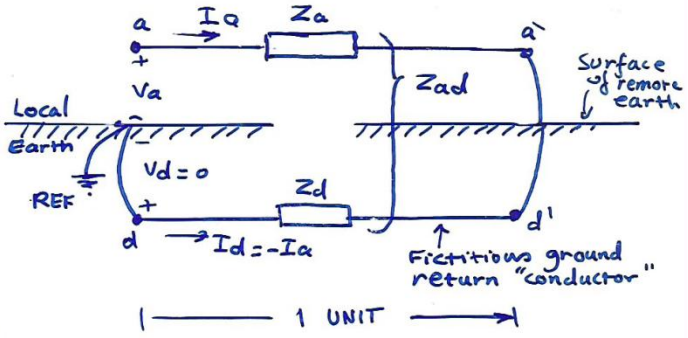
(iii) For Transmission lines:

In overhead T.L, Z_0 depends on the type of the line and the electrical conductivity of the soil. It is in general equal to:

Zero Sequence Reactance: $X_{0L} = X_{1L} \times (3 - 3.5)$ depending on the conductor cross-section and conductor arrangements.

* The effective (a.c.) resistance r_{do} ↑ of the zero-sequence impedance ($Z_o = r_{do} + jX_o$) includes both the effective resistance of the line conductor r_a plus the earth return path resistance r_d. i.e $r_{do} = r_a + r_d$

* Consider a single-conductor of one unit long and parallel to the ground. The conductor carries a current I_a with a return through circuit d-d' beneath the surface of the earth. The earth is considered to have a uniform resistivity and to be of infinite extent. The current I_a in the ground spreads out over a large area, seeking the lowest resistance return path and satifiying Kirchhoff's law to garantee an equal voltage drops in all pathes.



* Carson in 1923 found that the earth resistance r_d is a function of frequency and he derived the empirical formula in America and still be used till now.

$$r_d = 9.869 \times 10^{-4} f \quad \Omega/km.$$

for 50 Hz frequency

$$r_d = 0.0493 \ \Omega/km.$$ (This is for single line)

→ For 3-Lines $r_d = 3 \times 0.0493 = 0.1479 \ \Omega/km.$

* VDE Standards use the following empirical formula:

$$r_d \cong 0.15 \ \Omega/km.$$

For overhead lines with earth-wires part of the zero-sequence current also flows back through the earth-wires. The use of steel earth-wires does not significantly reduce the zero-sequence reactance, but there will be a considerable reduction when earth-wires with good conductivity, such as ACSR (Aluminume Conductor with reifforced steel core). or steel-cored copper wires are employed.

Fault Resistance:

The resistance of the fault is normally that of the arc and may be approximated by:

$$R_a(\Omega) = 44 \frac{(V \text{ in KV})}{I_f} \quad \text{for} \quad V < 110 \text{ KV}.$$

and $$R_a(\Omega) = 22 \frac{(V \text{ in KV})}{I_f} \quad \text{for} \quad V > 110 \text{ KV}.$$

For example, a 400KV line would have an arc resistance R_a with a fault current of 4KA which is found as follows:

$$R_a = 22 \frac{400}{4000} = 2.2 \ \Omega$$

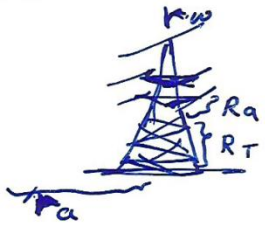
Hence for 200 km line; the resistance of the ground path is:

$$r'_d = 0.0493 \times 3 \times 200 = 29.58 \ \Omega.$$

Adding the fault resistance R_a:

$$r'_d + R_a = 29.58 + 2.2 = \underline{31.78} \ \Omega.$$

However the over all grounding resistance depends also on the footing resistance (resistance of the tower metal-work to ground) of the towers R_T, and also on the resistance per section of the ground wire r_w.



Normally R_T is taken as $\leq 10 \ \Omega$ with ground wire present.

Therefor we can say that the total ground resistance for the above example is

$$R_g = r'_d + R_a + R_T = 29.58 + 2.2 + 10 = \underline{41.78} \ \Omega.$$

Inclusion of Fault Impedance

In case of a bolted fault, $Z_f = 0$, whereas for an arcing fault, Z_f is the arc impedance.

In the case of a transmission line insulator flashover, Z_f includes the total fault impedance between the line and ground, including the impedance of the arc and transmission tower, as well as the tower footing if there are no neutral wires.

(1) For L-G fault:

Fault condition

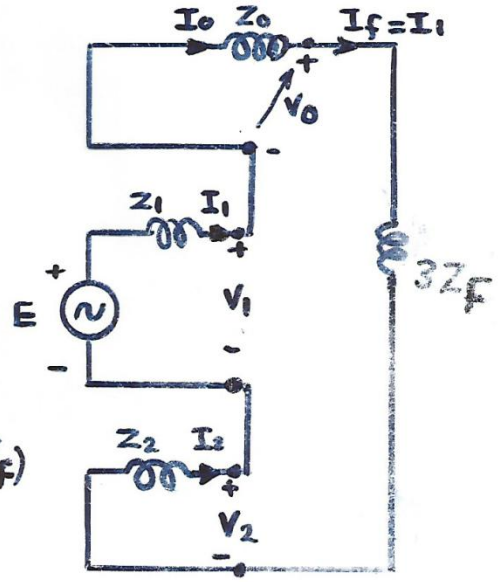
$I_b = I_c = 0$

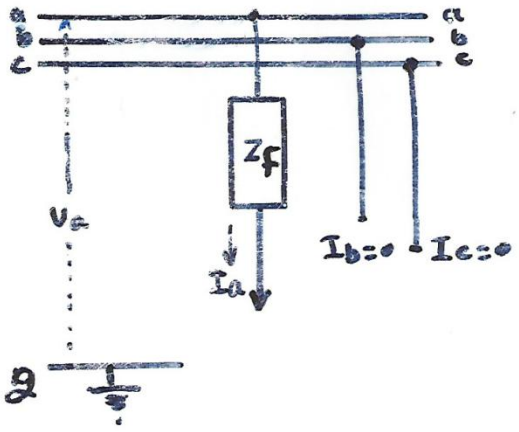
$V_{a-g} = Z_f I_a$

as $V_a = 3Z_f I_1$

$\therefore V_{a_0} + V_{a_1} + V_{a_2} = V_a = 3Z_f I_1$

$$I_0 = I_1 = I_2 = \frac{E}{Z_0 + Z_1 + Z_2 + (3Z_f)}$$

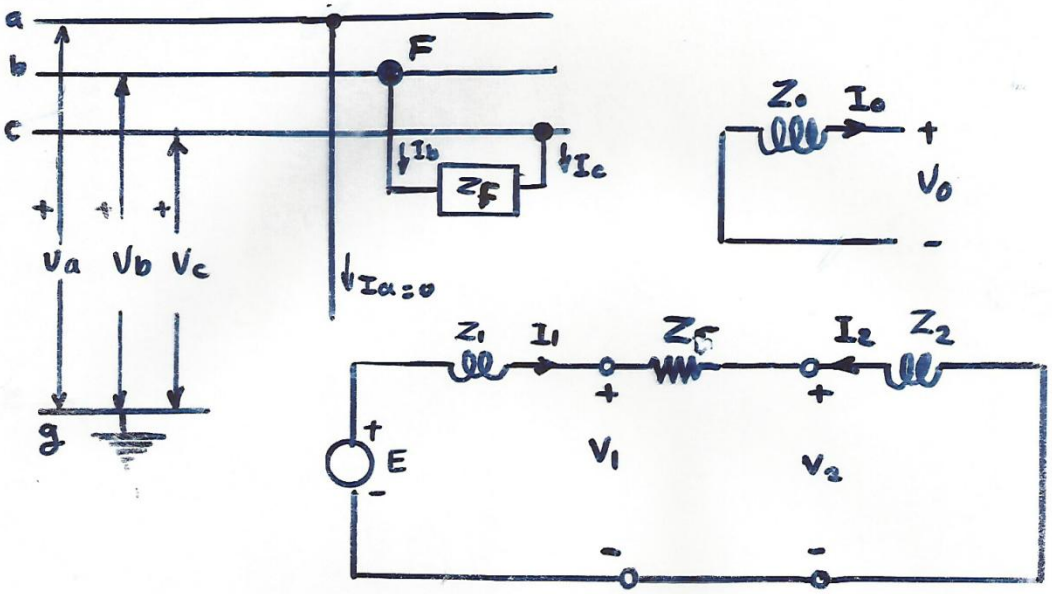




(2) L-L Fault (phase to phase fault):

Fault conditions: $I_a = 0$, $I_b = -I_c$

$$V_{bg} - V_{cg} = Z_f I_b$$



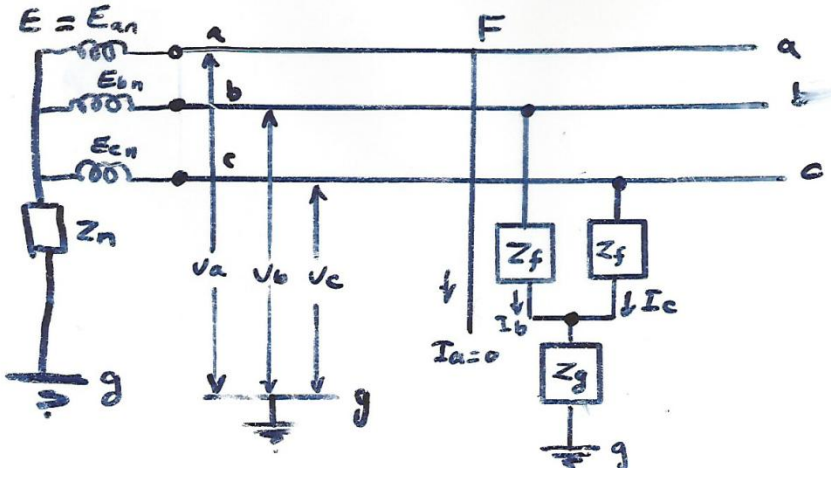
$$I_0 = 0$$

$$I_1 = -I_2$$

$$Z_f I_1 = V_1 - V_2$$

$$\therefore I_1 = \frac{E}{Z_1 + Z_2 + Z_f}$$

(3) Double line to ground fault (general case):



Fault conditions:

$I_a = 0$

$V_b = (Z_f + Z_g) I_b + Z_g I_c$

$V_c = Z_g I_b + (Z_f + Z_g) I_c$

or in terms of symmetrical components at the point of fault, these three quantities will be:

$$I_a = I_{a1} + I_{a2} + I_{a0} = 0 \quad \ldots(1)$$

$$V_b = \lambda^2 V_{a1} + \lambda V_{a2} + V_{a0} = (Z_f + Z_g)(\lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}) + Z_g(\lambda I_{a1} + \lambda^2 I_{a2} + I_{a0}) \quad \ldots(2)$$

$$V_c = \lambda V_{a1} + \lambda^2 V_{a2} + V_{a0} = Z_g(\lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}) + (Z_f + Z_g)(\lambda I_{a1} + \lambda^2 I_{a2} + I_{a0}) \quad \ldots(3)$$

subtracting eq. 3 from eq. 2 gives

$$(\lambda^2 - \lambda) V_{a1} + (\lambda - \lambda^2) V_{a2} = Z_f(\lambda^2 I_{a1} + \lambda I_{a2} + I_{a0}) - Z_f(\lambda I_{a1} + \lambda^2 I_{a2} + I_{a0})$$

$$(\lambda^2 - \lambda)(V_{a1} - Z_f I_{a1}) = (\lambda^2 - \lambda)(V_{a2} - Z_f I_{a2})$$

$$V_{a1} - Z_f I_{a1} = V_{a2} - Z_f I_{a2}.$$

Substituting this in Eq. 3 gives

$$\lambda(V_{a1} - Z_f I_{a1}) + \lambda^2 (V_{a1} - Z_f I_{a1}) + (V_{a0} - Z_f I_{a0}) = Z_g[(\lambda^2 + \lambda) I_{a1} + (\lambda^2 + \lambda) I_{a2} + 2 I_{a0}]$$

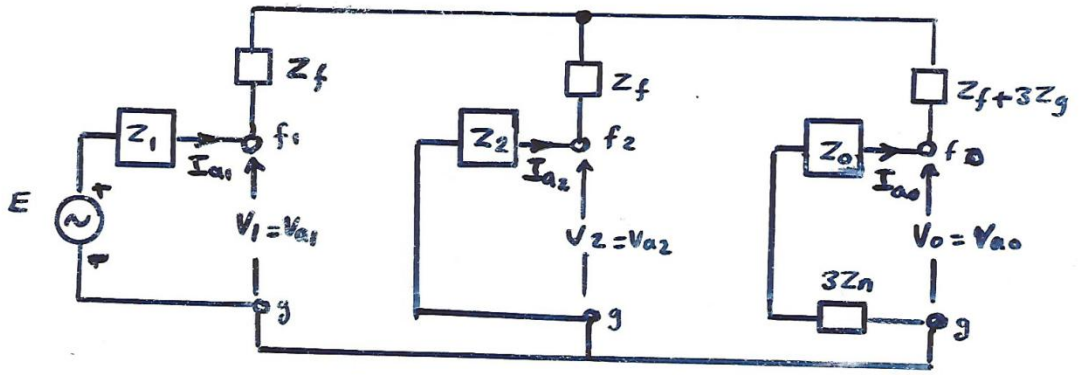
This is simplified to:

$$V_{a1} - Z_f I_{a1} = V_{a0} - Z_f I_{a0} - Z_g(-I_{a1} - I_{a2} + 2 I_{a0})$$

but from eq. (1): $(-I_{a1} - I_{a2}) = I_{a0}$

$$\therefore V_{a1} - Z_f I_{a1} = V_{a2} - Z_f I_{a2} = V_{a0} - (Z_f + 3Z_g) I_{a0} \quad \ldots(4)$$

Equations 1 & 4 are satisfied by the interconnection of the sequence networks shown in the following figures:



Interconnection of the sequence networks for a general case double-earth fault on b and c.

This is a general case of double-earth-fault on two phases (b & c).

From this general solution we can find L-L (phase-phase) fault let $Z_F = 2Z_f$ and $Z_g = \infty$.

and for L-L (phase-phase) fault without Z_f to ground.

let $Z_f = 0$ and $Z_g = 0$.

PHASE SHIFT IN 3-Phase TRANSFORMERS

There are several methods for transforming 3-ph voltages to higher or lower 3-ph voltages. However four main types of 3-ph transformers are in common use:

1. Wye - wye (Y-Y)
2. Wye - delta (Y-Δ)
3. Delta - Wye (Δ-Y)
4. Delta - Delta (Δ-Δ)

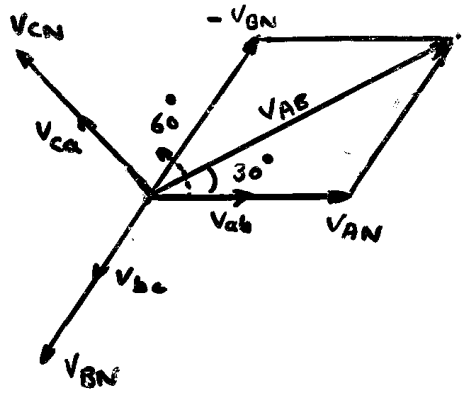
The Y-Y connection is seldom used because of possible voltage unbalances and probles with third harmonic voltages. However it is economical for small, high voltage transformers because the number of turns/phase and the amount of insulation is low (as phase voltage is only $\frac{1}{\sqrt{3}}$ of line voltage).

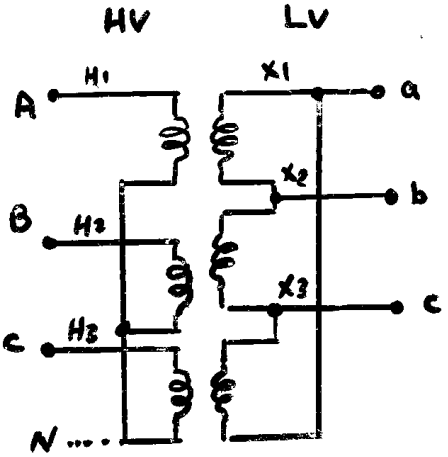
The Δ-Δ is economical for large, low-voltage transformers. It has a main advantage that one of the phases can be removed for repair or maintenace. The remaining two phases continue to function as 3-ph system (although the rating will be reduced to 58%.).

N.B <u>For connection Y-Y or Δ-Δ for primary and secondary there is no phase shift between them.</u>

The Y-Δ or Δ-Y connection there is a phase shift between primary and secondary of the transformer. To understand this problem, let us consider a Step-down transformer has Y-Δ connection as shown in Fig. 1.

Consider the Δ as the low voltage side, then from the study of the phasor diagram





$V_{AB} = 2 \times V_{AN} \cos \frac{60}{2} = 2 V_{AN} \cos 30^\circ = 2 V_{AN} \frac{\sqrt{3}}{2} = \sqrt{3} V_{AN}$

which leads V_{AN} by 30°. Hence:

$$V_{AB} = \sqrt{3} V_{AN} \angle 30^\circ .$$

& $$V_{ab} = \frac{\sqrt{3} V_{AN}}{K} \angle 0^\circ$$

where K is the transformation ratio

In fact all of high voltage magnitudes <u>lead</u> low voltages by 30°.

Δ-Y connection give same angular displacement which is 30°.

From many trying it can say that:

For each Y-Δ or Δ-Y named, the phasors as make positive sequence magnitudes in HV side lead more than positive sequence magnitudes in LV side by 30° and Vise Versa according to negative sequence magnitudes.

SYSTEM GROUNDING

Defenitions: The word "GROUNDING" is commonly used in electric power system work to cover both "system grounding" and "equipment grounding". These terms are defined by the National Electric code as follows:

A. System Ground: is a connection to ground from one of the current-carrying conductors of a distribution system or of an interior wiring system.

B. Equipment Ground: An equipment ground is a connection to ground from one or more of the non-current-carrying metal parts of the wiring system or of apparatus connected to the system.

The following definitions are taken from AIEE Standard No. 32 Neutral Grounding Devices:

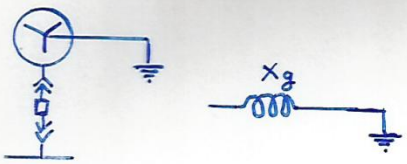
1. System Neutral Ground: is a connection to ground from the neutral point or points of a circuit, transformer, rotating machine, or system.
2. Grounded system: is a system of conductors in which at least one conductor or point (usually the neutral point of transformer or generator windings) is grounded, either solidly or through a limiting (current-limiting) device.
3. Solidly Grounded (Directly Grounded): means grounded through an adequate ground connection in which no impedance has been inserted intentionally.
4. Resistance Grounded: This is a grounding through impedance, the principal element of which is resistance
5. Reactance Grounded: This means grounded through impedance, the pricipal element of which is reactance.
6. Resonant Grounded (Tuned Grounded): This means reactance grounded through such values of reactance that, during a fault between one of the conductors and earth, the rated-frequency current flowing in the grounding reactances and the rated-frequency capacitance current flowing between the unfaulted conductors and earth

Shall be equal.

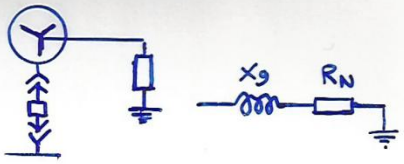
7. Ground - fault Neutralizer: A ground - fault neutralizer is a grounding device which provides an inductive component of current in a ground fault that is substantially equal to and therefore neutralizes the rated frequency capacitive component of the ground - fault current, thus rendering the system resonant grounded.

8. Grounding Transformer: A grounding transformer is a transformer intended primarily to provide a neutral point for grounding purposes.

METODS OF NEUTRAL GROUNDING:

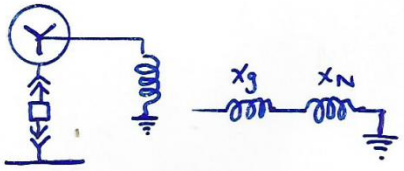


1. Solidly Grounded

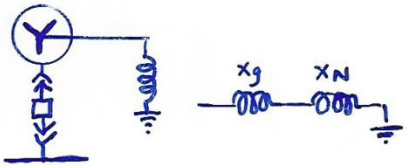


2. Resistance Grounding

R_N is one or more resistance

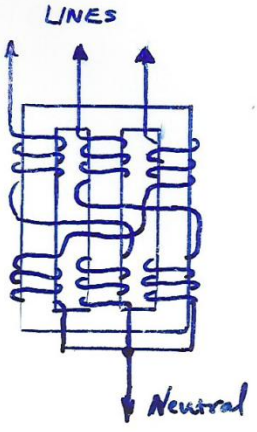


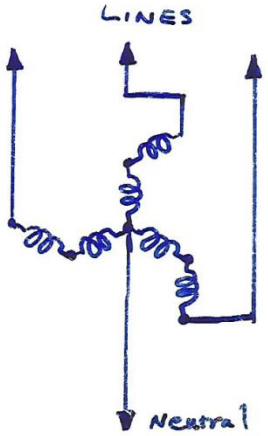
3. Reactance Grounding



4. Ground fault neutralizer.

ZIGZAG GROUNDING TRANSFORMER





In general, in case of unbalanced fault conditions the connection of the transformer and generator neutrals greatly influences the fault currents and voltages.

In most high-voltage systems the neutrals are solidly grounded. i.e. connected directly to the ground, with the exception of generators which are grounded through a resistance to limit stator fault currents.

Ordinary transformers neutral point may either be isolated or connected to ground through a resistance or reactance or solidly grounded.

Auto transformers used for interconnection of supply networks working at different voltages e.g. 132 KV to 400 KV. The neutral point should be solidly grounded and intervening resistance or reactance is not permitted.

For Δ connected transformers, a means of providing an earth point or neutral is obtained by using of a zigzag transformer. This is done by interconnection of two windings on each limb, a node of zero potential is obtained.

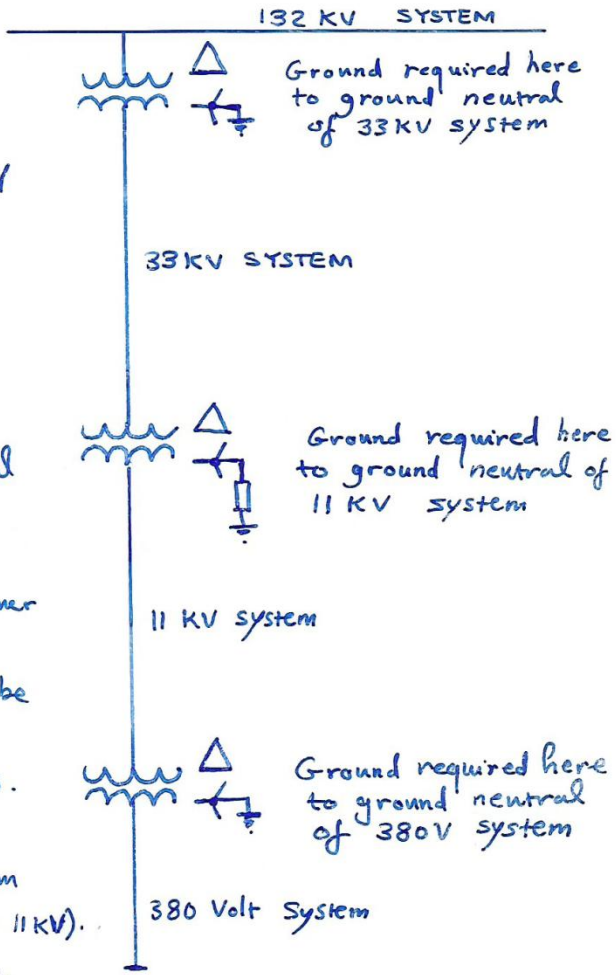
The advantages of solid grounding are as follows:

1. Voltages to ground are limited to the phase voltages.
2. Intermittent ground faults and high voltages due to arcing faults are eliminated.
3. Sensitive protective relays operated by earth fault currents clear these faults at an early stage.
4. Greater safty for personnel and equipment.

SELECTION OF SYSTEM GROUNDING POINT

GROUND AT EACH VOLTAGE LVELE.

It is necessary to ground each voltage level to achieve the protection and advantages of neutral grounding. This is illustrated in the following figure:


132 KV SYSTEM
Ground required here to ground neutral of 33KV system
33KV SYSTEM
Ground required here to ground neutral of 11 KV system
11 KV system
Ground required here to ground neutral of 380V system
380 Volt System

For example, if the 11KV system in this diagram is not grounded, this level would have all the characteristics of ungrounded system, at the same time the 33KV and 380 Volt levels would have the characteristics of grounded neutral system.

Each voltage level may be grounded at the neutral of: (a) a generator
(b) transformer
or (c) grounding transformer

N.B (1) The ground should be at the power sources and not at the loads.

(2) Resistance grounding only recomended for medium voltage systems (6.6Kv or 11KV).

(3) Solidly ground neutral is used for all systems above 20kv. (no rotating equipment assumed operating directly at these voltages).

(4) For low-voltage systems (220 – 600 V). The generator neutral must be grounded through low-value reactance. However transformers must be solidly grounded at the Y's of the secondaries. If Δ-Δ transformers are used

Grounding transformers must be connected on secondary sides.

Comparison of kinds of faults

1. Comparison between relation derived for $3-\phi$ fault and $L-L$ fault shows that for $L-L$ fault I_f does not exceed $\frac{\sqrt{3}}{2}$ (I_{sc} of the $3-\phi$ fault), when $Z_2 = Z_1$.

2. For $L-L-G$ & $L-G$ faults it is found that under certain conditions I_f may exceed the three-ϕ short circuit current I_{sc}. This may lead to considerable voltage rises on the healthy line conductors.

   Since the ground has considerable effect in these type of fault let us examine its effect as follows:

   (a) Network with isolated neutral

   It has been found that for $L-G$ & $L-L-G$ faults extremely high currents and voltages occur when the difference in impedance angle between Z_1 and Z_0 ($\phi_1 - \phi_0$) is large and the ratio Z_0/Z_1 is small.

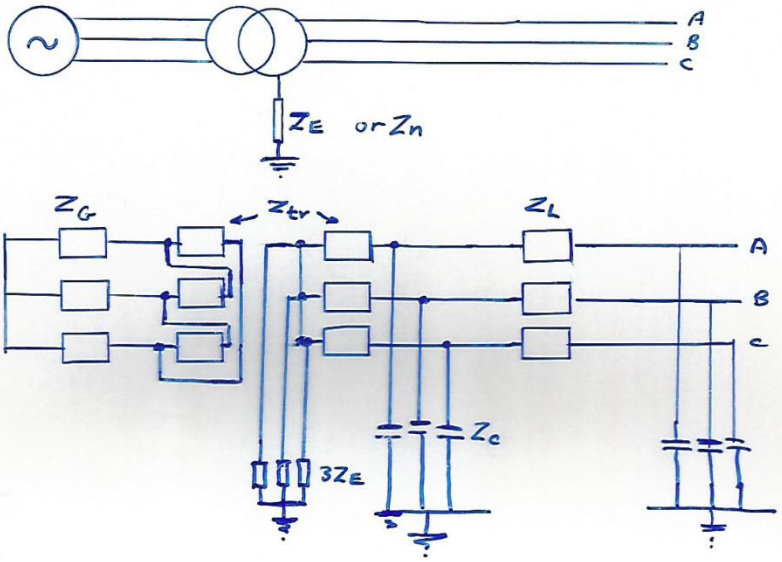
   However for networks with isolated neutral $\phi_1 - \phi_0$ will be large if we exclude the capacitive earthing of the system neutral.

   On the other hand, in order that Z_0/Z_1 in such a network to be small, the $3-\phi$ short circuit current at the fault point should not greatly exceed the charging current of the transmission lines metallically connected together in the network. But this case cannot occur in practice. So, generally, Z_0/Z_1 will be at least 5 and usually much greater.

   * It is also found that for $L-G$ & $L-L-G$ faults even when $Z_0/Z_1 \gg 5$, $I_f < I_{sc}$ for $3-\phi$ fault. Hence both kinds of fault may generally be ignored when calculating maximum short-circuit currents. Hence, in networks with isolated neutral the max. short circuit current is always that associated with a $3-\phi$ fault.

(b) Networks with inductively earthed neutral

Consider the following simple network:



Z_G short cct impedance of the generator per phase

Z_{tr} " " " " " transformer " "

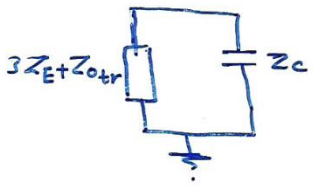
Z_L Impedance of the line per phase

Z_c Capacitive impedance of the line conductor to earth

Z_E Earthing impedance of the system neutral.

The earthing impedance Z_E (or Z_n) is common to all three-phases and is therefore included in the single-phase equivalent circuit with a value $3 \times Z_E$. $\left(\frac{1}{Z_E} = \frac{1}{3Z_E} + \frac{1}{3Z_E} + \frac{1}{3Z_E}\right)$

If the neutral is earthed through inductive reactance, so that we shall have the following circuit / phase



with the inductance $3Z_E + Z_{otr}$ exactly tuned to the network capacitance Z_c, the impedance Z_o of the system becomes ∞

Hence For L-G & L-L-G faults an almost zero current flows to earth. & L-L-G fault will be same as L-L fault clear of earth.

Also in this case the max. I_{sc} occur with 3-ϕ faults.

When the earthing inductance is not exactly tuned to the Z_C. two conditions arise:

(i) If $3Z_E + Z_{otr} > Z_C$ (undercompensation)

(ii) If $3Z_E + Z_{otr} < Z_C$ (over compensation)

In case (i) Z_o of the system will be of capacitive nature hence If in case of L-G & L-L-G fault will be less than that of network with isolated neutral.

In case (ii) Z_o will increase if only the inductive reactance $3Z_E + Z_{otr} > 50\% Z_C$. On this assumption If in case of L-G & L-L-G faults will also be less than that of network with isolated neutral.

If $3Z_E + Z_{otr} < 50\% Z_C$, Z_o will be reduced and will have inductive nature hence:

$\phi_1 - \phi_o$ will be in the range $0 - 90^\circ$

and $Z_o/Z_1 < 5$

Hence with L-G & L-L-G faults $I_f > I_{sc(3-\phi)}$ & I_f for L-L-G fault is greater than I_f for L-G fault.

(C) Networks with solidly earthed neutral:-

If the earthing impedance Z_E is reduced to zero, this will lead to solidly earthed network. Here Z_C will be short circuited since Z_{otr} & $Z_{OL} << Z_C$.

Here $\frac{Z_o}{Z_1} \simeq 0.5$ & $(\phi_1 - \phi_o) \overset{\text{range}}{\Rightarrow} 0 - 90^\circ$.

with L-G fault $I_f \simeq 1.3\ I_{sc(3-\phi)}$.

with L-L-G " $I_f \simeq 1.5\ I_{sc(3-\phi)}$.

However voltages on the healthy phases $\simeq 1.1$ Line-to-line system voltages.

Conclusion: (1) Highest short circuit current occurs with isolated neutral network with 3-ϕ faults.

(2) In networks earthed solidly or through a low inductance the highest s.c currents are with : L-G ,or L_L_G faults.

The disadvantages of Solid grounding are;

1. With a ground fault on one line does not place the remaining conductors at line voltage above ground as it is the case with isolated neutral.

2. Interference with telephone circuit is greatly increased because of the presence of zero-sequence currents.

Zero Sequence Impedance of a 3-ϕ 2-Winding Transformer:

1. The equivalent circuit of a Two-winding 3-ϕ transformer:

If a 3-ϕ transformer is supplied with balanced 3-ϕ voltages, and the load on it is balanced, then conditions in each phase are similar except for phase displacement. A single-phase equivalent circuit will be as shown in the following Figure;

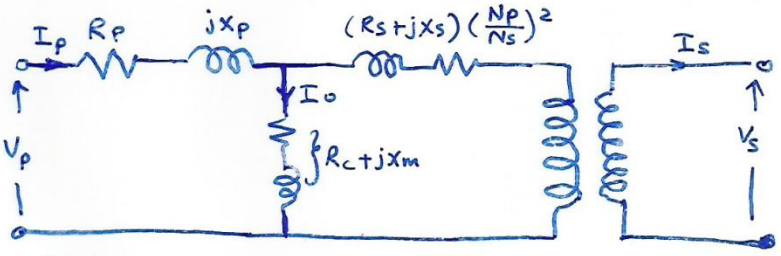


Fig.1 one-phase equivalent CCT

where: V_p = line to neutral-phase voltage applied to primary winding.

V_s = line to neutral-phase voltage at the secondary side.

I_p = primary current, I_s = secondary current

I_o = no load current.

R_p, R_s are the primary and secondary resistance respectively.

X_p , X_s are the primary and secondary leakage reactances/phase respectively.

$R_m + jX_m$ magnetizing impedance

N_p = number of primary turns/phase .
N_s = " " secondary " " .

Note: (1) for positive and negative sequences, the magnetizing (exciting) impedance may be neglected since it is very big compared with the positive or negative sequence impedance $Z = (R + jX)$, (about 25 p.u. compared with 0.1 p.u.) where $Z_1 = Z_2 = R + jX$. The impedances Z_1 and Z_2 of the transformer are equal to the values shown in the above figure, except for the 30° phase shift for Δ-Y transformer being oposite in sign for negative sequence to that for positive sequence.
($Z = R + jX$) is the total p.u. series positive-seq. impedance of one phase of both primary and secondary windings included together.
(2) In the steady-state analysis it is always assume Y-Y connected transformer for simplicity to avoid phase-shift between primary and secondary sides. But in Fault analysis this is not always the case and consideration must be taken for the transformer connections and earthing.

However, the impedance to zero-sequence current Z_0 may be about equal to the positive-sequence value or it may be infinity. ($Z_0 \simeq Z_1$ or ∞). This depends on:

(1) winding connections (Δ-Y, Y-Y, Y-Δ .....).
(2) Whether or not star points are grounded.
(3) type of magnetic circuit (linear, nonlinear). transformer construction: shell-type, 5 limb-core type, 3 limb core-type or 3 single-phase bank-type). This will effect the value of the magnetizing (exciting) impedance.

For shell-type or for 5 limb-core type or for 3-phase bank $Z_{0m} \simeq Z_{1m}$.

for 3-limb core type unit $Z_{om} << Z_{1en}$ (0.3-4 p.u.)

This is because the zero-sequence fluxes which are in phase with each other in all three limbs as shown in Fig. 2 have no return paths in the iron core, and must pass through air, copper, insulation and the tank wall. The zero-sequence flux is therefore limited in value.

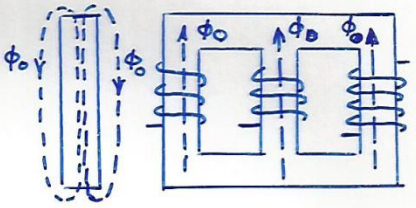


Fig. 2 Zero-sequence flux in 3-limb core-type Transformer.

Effect of Connection of Transformer windings

Apart from construction effect upon the zero-sequence exciting impedance, there is a relatively great effect on the windings connection. Fig. 3 shows a two-winding transformer equivalent circuit for which Z_1 is the total p.u. series positive-sequence impedance of one phase ($Z_1 = Z_2 = R + jX$), and Z_{om} is the p.u. zero-sequence shunt (exciting) impedance. Various cases will now be considered.

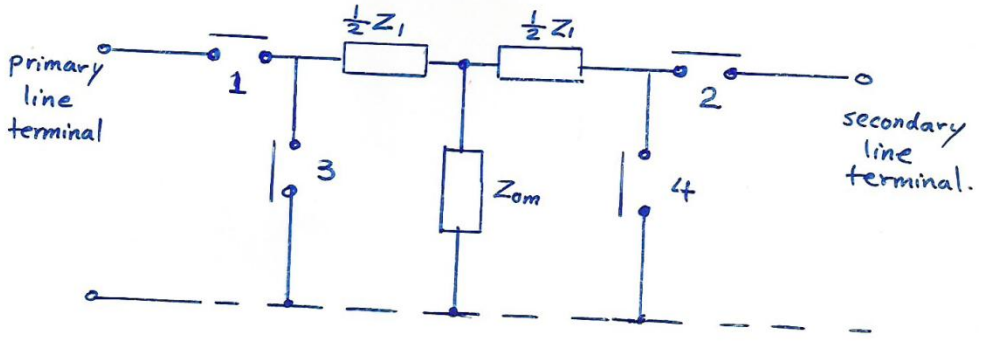


Fig. 3. Zero-sequence equivalent cct. For a transformer.

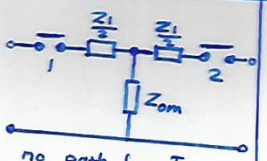
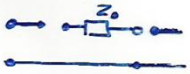
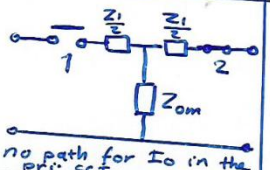
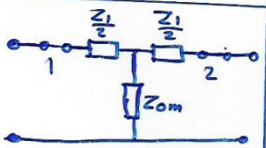
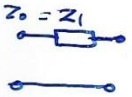
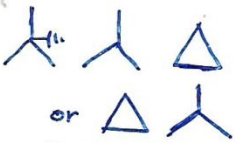
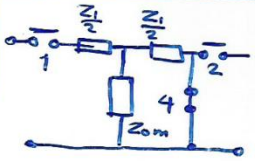
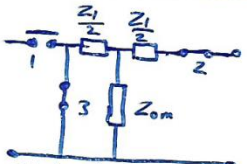
For a transformer.

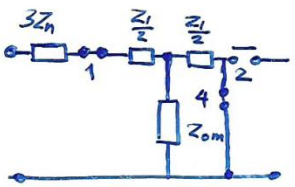
1. If the star-point on the primary side is earthed (grounded) then link (1) is closed.

2. If the secondary star-point is earthed link (2) is closed.

3. If the primary winding is delata-connected, link (3) is closed but link (1) is open.

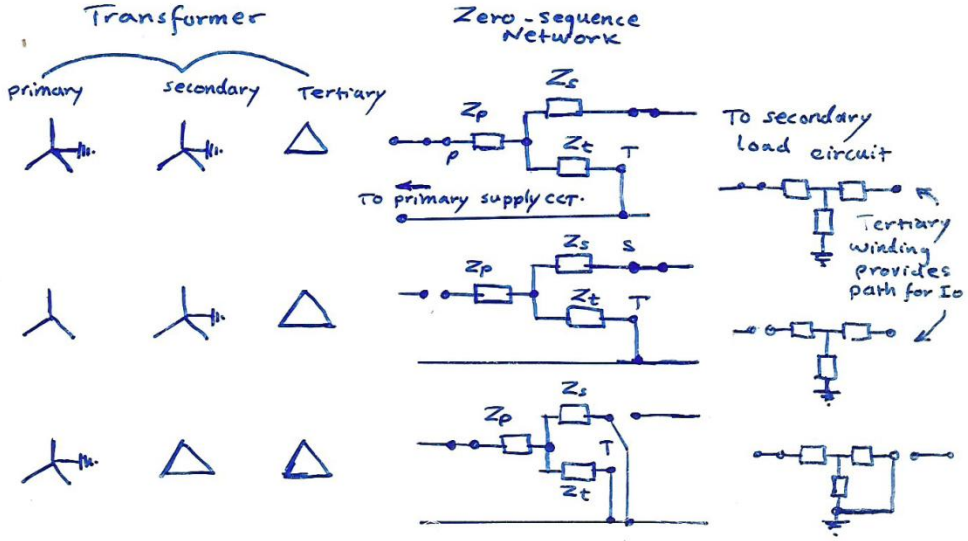
4. If the secondary is delta-connected link (4) is closed but 2 is open.

Generator	Transformer pri. sec.	Transformer Zero-Sequence Imp. network	Transformer Zero-sequence Imp.
		 no path for I_0 in the prim. & sec. cct.	$Z_0 = \infty$ on both sides This is applies equally wether the generator star point is earthed or not 
		 no path for I_0 in the pri. cct.	$Z_0 = \infty$ on the primary side $Z_0 = \frac{Z_1}{2} + Z_{om}$ on the secondary side (where Z_{om} is higher for shell, 3-seperate units or 5-limb core type than 3-limb core-type.
			$Z_0 = Z_1$ ≃  $Z_0 = Z_1 = Z_1$ for shell, or separate single-phase units $Z_0 \simeq 0.85 Z_1$ for core type. I_0 free to flow in pri. & sec. cct
 or			$Z_0 = \infty$ on both primary and secondary sides no flow of I_0 possible
		 Single-phase currents can circulate in	$Z_0 = \infty$ on the primary side $Z_0 \simeq Z_1$ on secondary side.

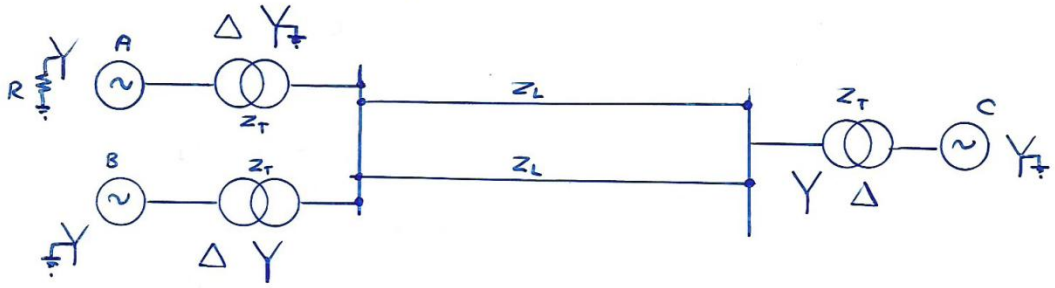
		$Z_0 = \infty$ on secondary side $Z_0 = Z_1$ on primary side, provided that generator star point is grounded. $3Z_n$ is included with Z_0 of supply circuit.
		No flow of I_0 possible

THREE-WINDING TRANSFORMERS

In case of 3-winding transformers, there is the possibility that zero-sequence currents can be caused to flow in the tertiary winding, due to the circuit to which it is connected. If the tertiary winding is delta-connected, then the zero-sequence circuit e.m.f induced in it can cause current to flow, and the tertiary impedance appears in the zero-sequence network. Typical examples are listed below where Z_p, Z_s and Z_t are the positive-(or negative) sequence impedances of the primary, secondary and tertiary windins respectively.

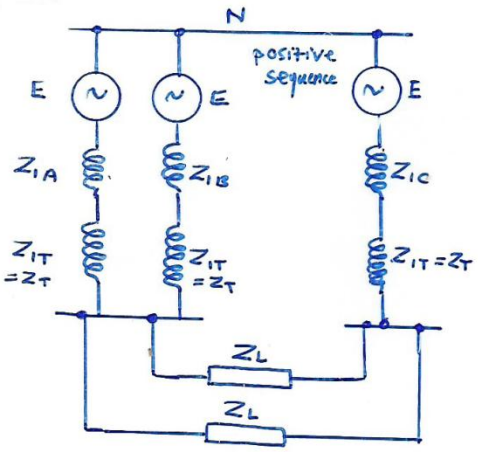


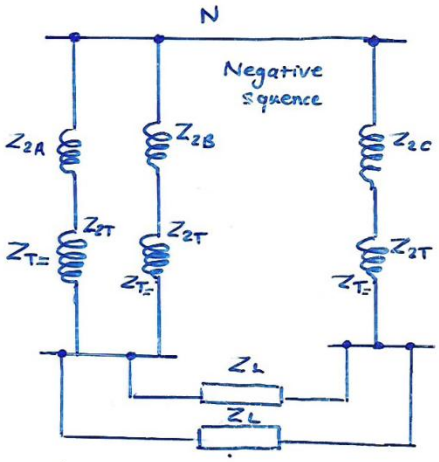
Example:- For the following simple network, draw the positive, negative sequence networks.

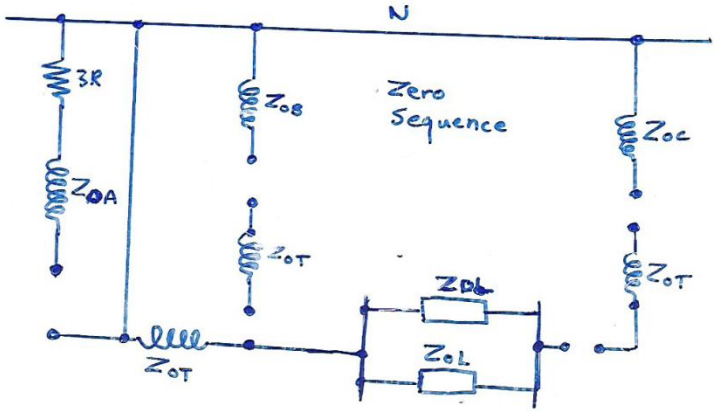


Network line diagram

Solution







SHORT CIRCUIT CALCULATIONS

PART I

Organized by :

EAST Institute For Human Resources Development

For the Period 31-3/5-6/1999- Baghdad

1. Introduction

Faults in three-phase system may be classified under the following headings:

1- Balanced three phase faults.

2- Single line - to - ground fault.

3- Line - to - line faults.

4- Double line - to ground faults.

Note that three out of four of these fault .types involve line - to - ground (Earth faults) and most of these are a result of insulator flash over due to several reasons involving weather condition and insulation quality. The balanced three phase fault is the rarest in occurrence accounting to above five percent of the total faults and it is the least complex of all types of short circuit studies is sofar as the calculations are concerned .The last three unsymmetrical studies will require the knowledge and use of tools of symmetrical components .Thus the first part of this presentation will deal with the balanced and unbalanced fault calculation in general and conventional methods .For small systems. .The second part will deal with large system using digital technique approach .

2. Symmetrical Fault analysis

In order to make correct choice in switch gear rating , it is necessary to perform the short - circuit calculations .Such calculation

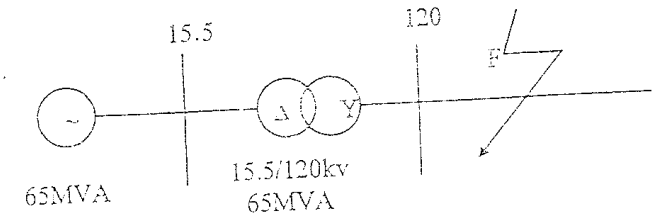
enable the fault MVA due to a symmetrical 3-phase fault at a point of interest to be determined . These calculation demand the reduction of the network to that of a single source feeding a single impedance.

The generator reactance's are normally taken as their sub- transient values in order to consider the most pessimistic conditions. Calculation may be performed by expressing impedance's either in ohms or in per - unit (pu) values. When transformers are involved all impedance's must be referred to a selected voltage base .This could be either primary, secondary of the transformer or any other selected voltage base .

Example:A 65 MVA , Y connected , 15.5 kv synchronous generator is connected to 15.5 kv to 120 kv , 65 MVA Δ-Y transformer -The sub-transient reactance X''_d of the machine is 0.12 pu on a 65 MVA base while the transformer reactance 0.10 pu on the same base , the machine is unloaded when a symmetrical fault is suddenly placed at point F as shown in Fig(2) .

(a) find the sub-transient symmetrical fault in per unit amperes and actual amperes on both sides of the transformer .

(b) what is the max. value possible of the dc current component and what does the total current approach as a max.in pu and instantanuos



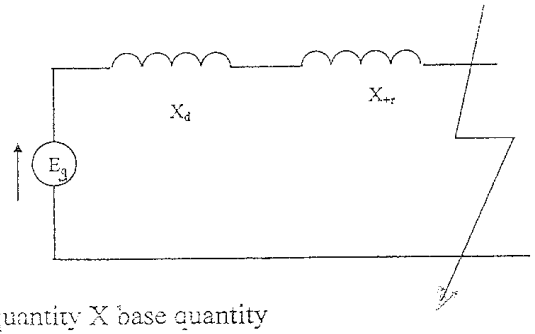
X"=0.12pu

Solution: (using the internal voltage method), here since the impedance's are based on the same MVA=65 therefor , there is no need to refer all

impedance to a selected base MVA by multiplying the impedance with a factor $= \frac{MVA_{base}}{MVA_{equipment}}$ .using pu values :

$$pu \quad I''_f = \frac{E_g}{X''_d + X_{tr}} = \frac{1.0\ pu}{j0.12 + j0.10}$$

$$= \frac{1.0}{j0.22} = 4.54 \quad \angle -90^\circ$$



Since Actual Quantity = pu quantity X base quantity

∴ To found the actual current we have to calculate the base current $(= \frac{\text{Base MVA}}{\sqrt{3}.\text{Base Voltage}})$

(a_1) Fault current on the 15.5kv side of transformer here the base kv=15.5kv

∴ base current $\frac{65 \times 1000}{\sqrt{3} \times 15.5} = 2420..\text{Amps}$

Actual current on the primary side =2420 * 4.54 =10990 Am ≅ 11 kA

(a_2) Fault current on the 120 kv side of the transformer :

here the base current $= \frac{65000}{\sqrt{3} \times 120} = 313$ Amp

Actual current on this side = 4.54 *313 =1420 A

Solution (b):-

$$pu \quad I_{dc}\ (\max) = \sqrt{2} \times rms \text{ value of } I_f$$

$$= \sqrt{2} \times 4.54 = 6.43 \text{ pu Am}$$

Max instantaneous us I (dc + symmetrical I)

$$\text{Max } I_{ot} = 2\sqrt{2} \times I_f$$

$$= 12.86 \text{ pu Amps}$$

Note: it was required to find the fault MVA or MVA level at F then :

$$(MVA)_{Fault} = \frac{(MVA)_{base}}{pu\ Z} = \frac{65}{0.22} = 295.1 \text{ MVA}$$

To find the fault current at F or fault current on the secondary of the transformer:

$$I_f = \frac{295 \times 1000}{\sqrt{3} \times 120} = 1420 \text{ Amp}$$

This is the most straight forward method of calculations:

1. Choose base MVA .
2. Refer impedance's to the base MVA.
3. Find equivalent single impedance
4. Use Formula to find (MVA) fault.

Example 2:

Fig(2) shows 4 bus network of 33 kv system if a symmetrical three-phase fault takes place at bus (4) .the system data are as follows:

Generators G1 and G2 both 25 MVA , 0.1 pu synchronous motor M ,20MVA , 0.1 pu Transmission lines , 12 , 23 , 34 , 13 , 24 are 10 Ω calculate the short circuit level at bus 4:

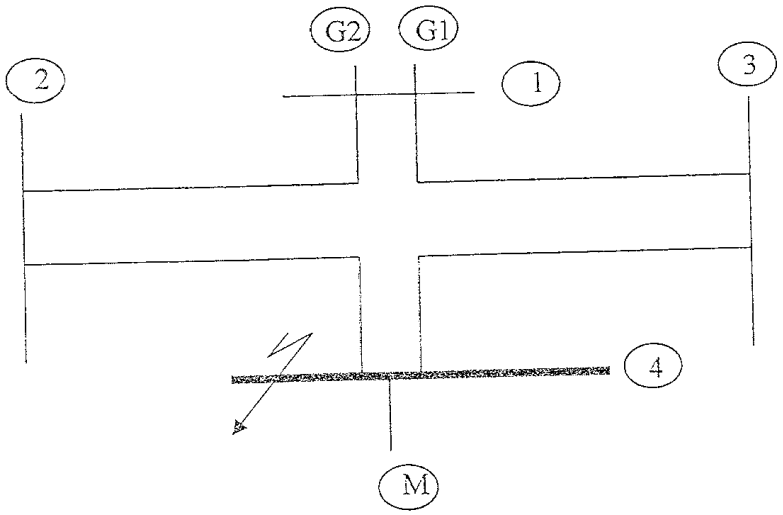


Fig (3) power system of example (2)

Solution :

Select base MVA = 100

$Z_{pu} = Z\Omega * (MVA_{base}/(kv)^2)$

$Z_L = 0.92$ pu

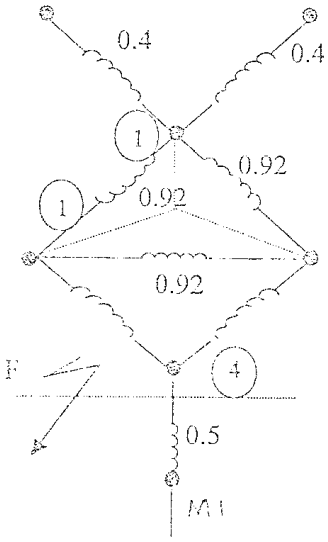
(p.u. impedance of each line = $10 \times \frac{100}{(33)^2}$

The impedance's of generator G1 and G2 referred to 100 MVA base :

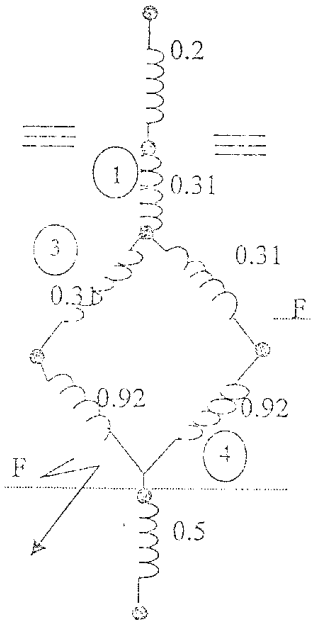
$$Z_{g_1} = Z_{g2} = 0.1 \times \frac{100}{25} = 0.4 \text{ pu each}$$

$$Z_{motor} = 0.1 \times \frac{100}{20} = 05 \text{ pu}$$

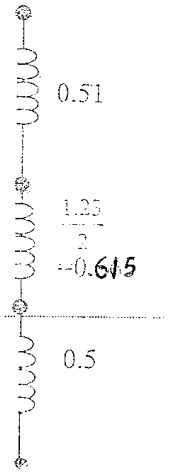
The reactance's is given by Fig(3) a :



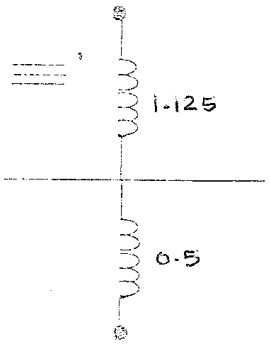
Fig(4)a



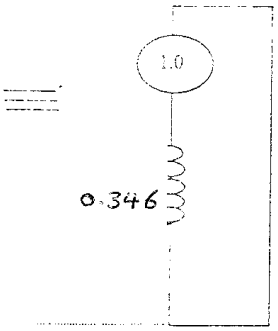
Fig(4)b



Fig(4)c



Fig(4)d



Fig(4)e

$\therefore$ Short circuit level $= \frac{100}{0.346} = 289$

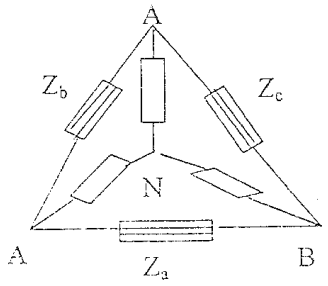
At B/B - 4

$(VA)_{f_{p.u}} = \frac{E_b}{X_b} = \frac{1}{X_b}$

S.C. Level $= (VA)_{f_{p.u}} \times$ BASE MVA

Summary of solution steps :

1. t is essential to select a base MVA (= 100) .
2. Refer all the impedance's to the selected base .
3. Draw the reactance network as shown Fig(3) a
4. The two generators are in parallel and equivalent reactance is 0.2 [Fig(3)b]
5. for the reactance forming the delta 1,2,3 us Δ-Y transformation



$$Z_{AN} = \frac{Z_b . Z_c}{Z_a + Z_b + Z_c}$$

$$Z_{BN} = \frac{Z_c + Z_a}{Z_a + Z_b Z_c}$$

$$Z_{CN} = \frac{Z_a + Z_b}{Z_a + Z_b + Z_c}$$

Fig(5)

6) Simplify Fig (3) b → fIg(3)c → Fig (3)d →Fig(3) e

This procedure of transformation and simple fraction is known as network reduction method

7) short circuit $= \frac{\text{Base MVA}}{\text{pu .reactance}} = \frac{100}{0.346}$

$= 289$MVA at the point of fault i.e bus (4)

Note (1) The synchronous motor is consider as supplying the network with reactive power as a generator as considered above .

Note (2) The short circuit MVA is a reactive power as the resistance's were neglected this is acceptable assumption because $X >> r$.

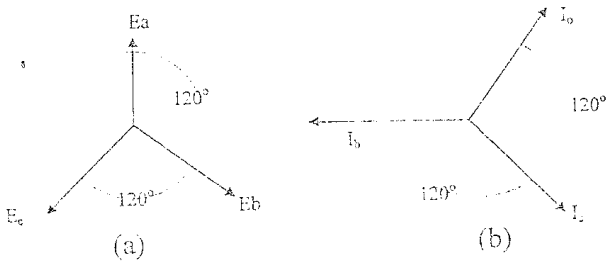
3. Un symmetrical Faults

As was stated earlier the majority of system faults which occur in practice are the in balanced or un symmetric faults.

An in balanced power system may be defined as a system in which the current or voltages are unbalanced .It will be evident that such an unbalanced may be brought about if either the alternator voltage are unsymmetrical or if the circuit corresponding to the system is so .

3.1 Symmetrical components :

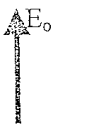
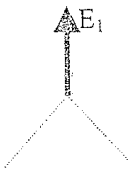
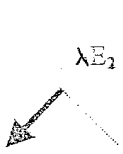
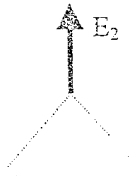
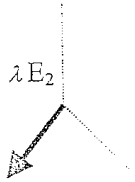
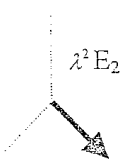
First it necessary to recall the concept of "phase sequence" of the voltages and the current of three phase system .By this is meant the order in which these quantities assume their maximum values .For example , for the voltage represented in fig(6) a, the phase



Fig(6)

order is a, b, c , a, i.e positive order (positive sequence for the current vectors the order is a, c, b, a, negative order (Negative sequence).

Assuming the reference voltage of the positive and negative sequences system to be those of phase α and introducing the operator λ to express the sequence voltage, the other phase with respect to the reference .

Sequence Component		Phase Vectors		
		A	B	C
	0			
	1			
	2			

For each phase the total value of the vector is equal to the three components $E_a = E_o + E_1 + E_2$ (reference vector)

$$E_b = E_o + \lambda^2 E_1 + \lambda E_2$$

$$E_c = E_o + \lambda E_1 + \lambda^2 E_2$$

$\lambda = -0.5 + j\,0.866$

$\lambda^2 = -0.5 - j\,0.866$

in the matrix from :

$$\begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} E_o \\ E_1 \\ E_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

we will define the operator matrix [P] where :

$$[p] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

the sequence component matrix , which we are often is

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = [P]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{.....(4)}$$

taking the inverse of [P] using the conventional procedure given in matrix algebra

$$P^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \quad \text{.....(5)}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \text{.....(6)}$$

or written as separate equations :

$$V_0 = \frac{1}{3}(V_a + V_b + V_c)$$

$$V_1 = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_c) \quad \text{.....(7)}$$

$$V2 = \frac{1}{3}(Va + \lambda^2 V_b + \lambda V_c)$$

similarly as equation (5) and (6) we can write the relation of phase current in term of sequence current and vice versa , as follows :

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda^2 & \lambda \\ 1 & \lambda & \lambda^2 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad \text{.....(8)}$$

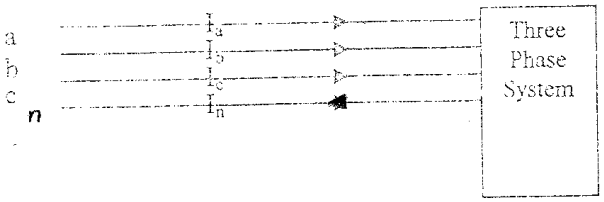
$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \text{.....(9)}$$

The general nature of zero - sequence current

From eq'n (9) $I_0 = \frac{1}{3}(I_a + I_b + I_c)$ and referring to fig (7) , since $I_n = I_a + I_b + I_c$

then :

$$I_n = 3I_0 \quad \text{.....(10)}$$



Fig(7)

we conclude that when ever we have a three - wire , 3 phase system without a neutral return , the zero sequence component of the line current is always zero .

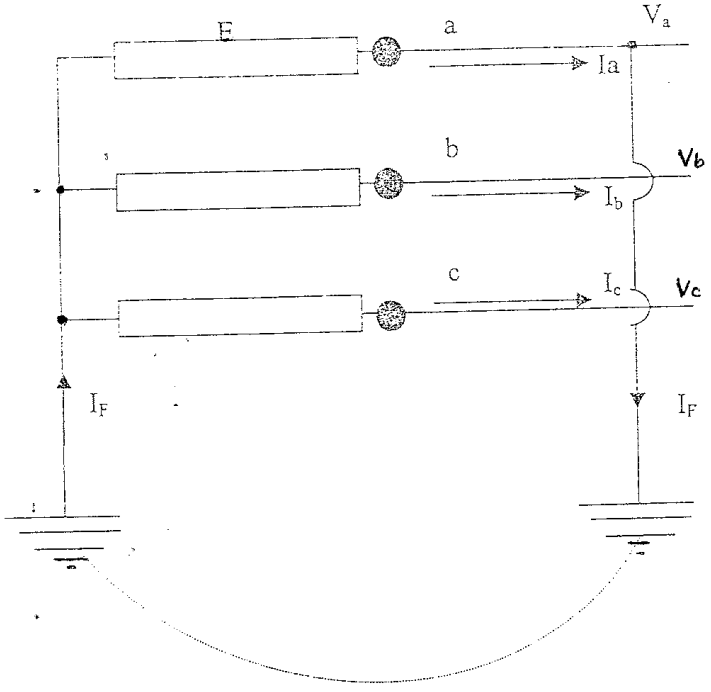
- In practical , lines outgoing of a three wire y-connection or Δ connection cannot carry zero - sequence currents . However zero-sequence current may exist in a four - wire system . They may also be found to circular within the delta of a transformer . This is possible , since its known that zero - sequence current of phases a,b and c are equal in magnitude and in phase with one another .

3.2 Analysis of Fault current by symmetrical components :

There are three ways in which unsymmetrical faults may occur in power system , they are :

(1) Single - phase to earth fault.
(2) Phase to phase fault.
(3) Double - phase to earth fault.

3.3 Single phase to earth fault



Fig(8)

The single phase - to - earth fault is charaterised by three conolitions:

$V_a = 0$ connected to earth

$I_b = 0$

$I_c = 0$

from the equation (9) and using the above condition

$I_0 = \frac{1}{3}(I_a + I_b + I_c)$

$I_1 = \frac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$

$I_2 = \frac{1}{3}(I_a + \lambda^2 I_b + \lambda I_c)$

$I_b = I_c = 0$ gives

$I_0 = I_1 = I_2 = \frac{I_a}{3}$ .....(10)

using the condition $V_a = 0$ let Z_1 the positive phase sequence impedance

Z_2 be the negative phase sequence impedance.

Z_0 be the zero phase sequence impedance .

But the voltage $V_a = E -$ (total sequence voltage drop) .

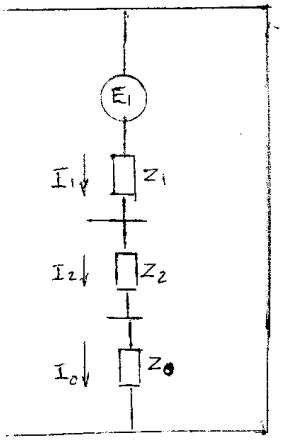
$$V_a = E - (I_1 Z_1 + I_2 Z_2 + I_0 Z_0)$$

but $I_1 = I_2 = I_0$ (take I_1)

$$V_a = E - I_1(Z_1 + Z_2 + Z_0)$$

$$I_1 = \frac{E}{Z_1 + Z_2 + Z_0} \quad \text{.....(11)}$$

$$I_a = \frac{3E}{Z_1 + Z_2 + Z_0} \quad \text{.....(12)}$$



the emf of phase b is given by $\lambda^2 E$

$$I_b = I_0 + \lambda^2 I_1 + \lambda I_2$$

$$V_b = \lambda^2 E - I_0 Z_0 - \lambda^2 I_1 Z_1 - \lambda I_2 Z_2$$

and $V_b = -E\frac{3Z_0 - j\sqrt{3}(2Z_2 + Z_0)}{2(Z_1 + Z_2 + Z_0)}$

It is evident that the sequence currents are obtained simply by connecting the phase - sequence impedances in series as shown in

Figure above.

Example (3):

a 3- phase , 33 kv , 37.5 MVA alternator is connected to a 33-kv overhead line which develops an earl· fault on one connector at the remote end . the positive , negative and zero- sequence reactance of the alternator are , 18,12 and 10% (on rating) which those fore the line are 6.3 ,6.3 and 12.3 Ω/conductor . Calculate the fault , current , and the phase voltages at the alternat terminals . Assume the alternator star point is solidly earthed.

Solution:

Base 37MVA (3-phase) , 33 kv (line)

Base voltage =1.0 pu = $\frac{33}{\sqrt{3}} = 19.1$ kv/ph

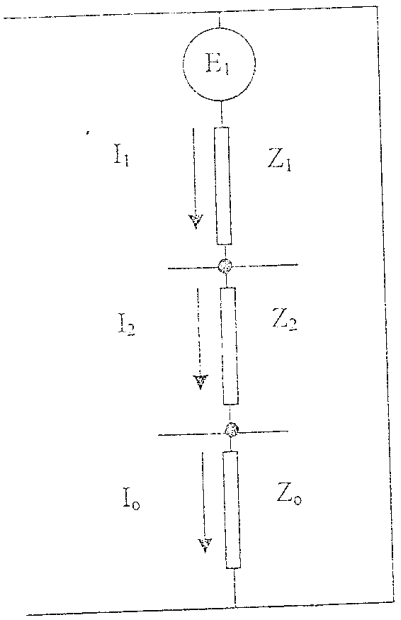
Base current = 1.0 pu = $\frac{37.5}{\sqrt{3} \times 33} = 0.655$ kA

Base reactance = 1.0 pu = $\frac{19.1}{0.655} = 29.1\Omega$

X_1 and X_2 of the line is 0.2165 p.u.
and $12.6 = \frac{12.6}{29.1} = 0.433$ p.u.
which is X_0 of the line.

so 6.3 ohms = $\frac{6.3}{29.1} = 0.2165$ pu (and of the line the total reactance of the equivalent network of the three sequence networks in series Fig (9) .

It will be evident that the sequence currents are obtained simply by connecting the phase - sequence impedances in series as shown in fig(9).



Total system $X_1 = 0.3965pu$

Total system $X_2 = 0.3360pu$

Total system $X_0 = 0.5330pu$

Total $X_t = (Z_1 + Z_2 + Z_0)$

$= 0.3965 + 0.3336 + 0.532$

$= 1.2655$ pu

$$I_1 = \frac{1.0\angle 0^0}{j1.266} = -j0.791pu$$

$$I_f = -j0.791 \times 3 = 2.37pu$$

Actual I_f in Amps = pu I_f

= 2.37 X 0.655= -j 1.55kv

The voltage with respect to earth at the a

from:

$$V_1 = E_1 - I_1 Z_1$$
$$V_2 = -I_2 Z_2$$
$$V_0 = I_0 Z_0$$

where the reactance are those from the s

$V_1 = 1.0\angle 0^0 - (-0.791)(+j0.18) = 0.858 + j0pu$

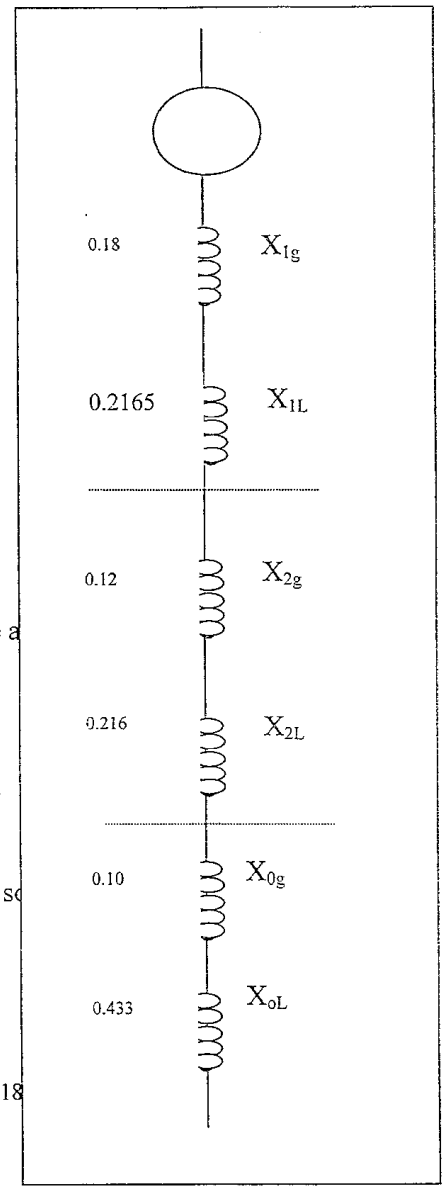
$V_2 = -(0.791)(+j0.12) = -0.0948 + j0pu$

$V_0 = -(-0.791)(+j0.1) = -0.0791 + j0pu$

$V_a = V_1 + V_2 + V_0 = 0.6481\angle 0pu = 13\angle 0kv$

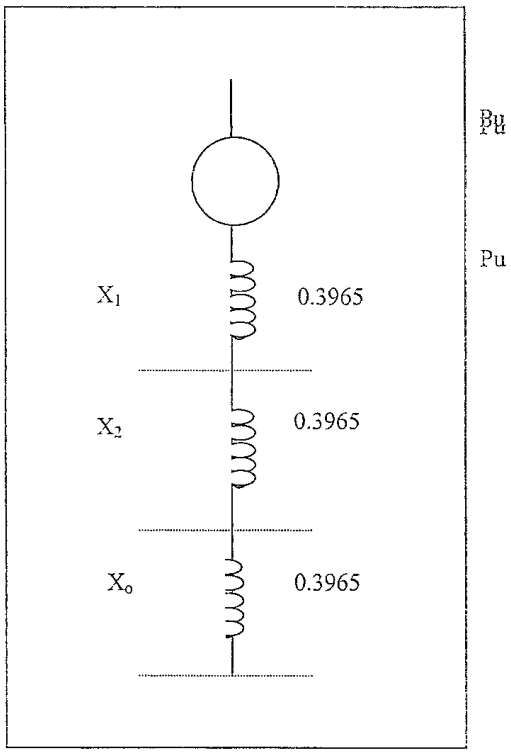
$V_b = V_0 + \lambda^2 V_1 + \lambda V_2 = 0.945\angle -119.16pu = 18$

$V_c = 18\angle +119.16kv$

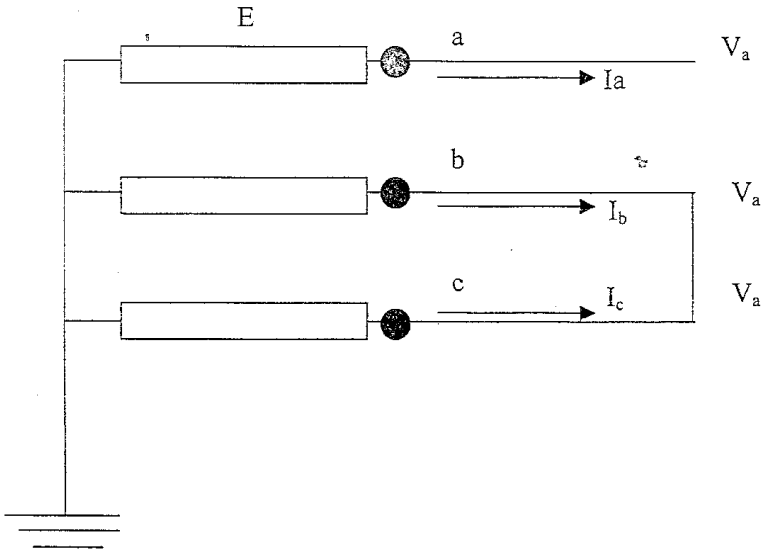


The fault MVA = 1.55 X 19.1 = 29.6 MVA

Note in practice the alternator are usually earthed via a resistor of value about 1.0 pu on alternator rating . The corresponding value to be added to x_t is 3 pu.



3.2 Phase to phase fault :



Condition of fault are

$I_a = 0$

$I_c = -I_b$

$V_b = V_c$

From which the three sequence currents_I_1, I_2, and, I_0 can be determined .

From equ'n (9) .

$$I_0 = \frac{1}{3}(I_a + I_b + I_c)$$

$$= \frac{1}{3}(0 + I_b - I_b) = 0$$

$I_0 = 0$ This is expected from the general nature of zero - sequence current as the fault does not involve earth or neutral wire , i.e no path for sequence currents .

again

$$I_1 = \frac{1}{3}(I_a + \lambda I_b + \lambda^2 I_c)$$

$$= \frac{1}{3}(0 + \lambda I_b - \lambda^2 I_b)$$

$$= \frac{1}{3} I_b(\lambda - \lambda^2)$$

$$I_2 = \frac{1}{3} I_b(\lambda^2 - \lambda)$$

i.e. $I_1 = -I_2$

again $V_b = V_c$ and express by impedance voltages

$$\lambda E^2 - \lambda^2 I_1 Z_1 - \lambda I_2 Z_2 = \lambda E - \lambda I_1 Z_1 - \lambda^2 I_2 Z_2$$

$$E(\lambda^2 - \lambda) = I_1[Z_1(\lambda^2 - \lambda) + Z_2(\lambda^2 - \lambda)]$$

$$E = I_1 Z_1 + I_1 Z_2 = I_1(Z_1 + Z_2)$$

$$\therefore I_1 = \frac{E}{Z_1 + Z_2}$$

Note :

To demonstrate how we treat the same case using matrix manpulation as follows :

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_0 = 0$$

$$I_1 = (\lambda - \lambda^2)I_b$$

$$I_2 = (\lambda^2 - \lambda)I_b$$

$$\therefore I_1 = -I_2$$

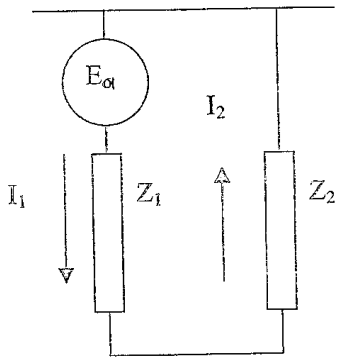
$$V_1 = \frac{1}{3}(V_a + \lambda V_b + \lambda^2 V_b)$$

$$V_2 = \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_b)$$

$$\therefore V_1 = V_2$$

$$\begin{bmatrix} 0 \\ V_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} 0 \\ I_1 \\ -I_1 \end{bmatrix}$$

$$\therefore I_1 = \frac{Ea}{Z_1 + Z_2}$$



Fig(11)

the interconnection of the sequence networks for simulating phase - to - phase fault is shown in Fig (11) .

Example (4)

A 25 MVA , 13.2 kv alternator with solidly earthed neutral has a subtransient reactance of 0.25 pu . The negative and zero sequence reactance's are 0.35 and 0.1 pu respectively . A Phase - to phase fault occurs at the terminals of a unloaded alternator. Determine the fault current and the line to line voltage at the fault.

Solution:

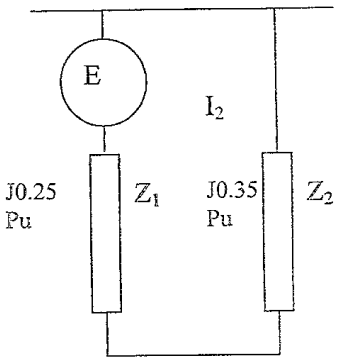
The sequence network & 0 L - L fault is shown in fig (11) since the zero sequence network is absent . assuming 1.0 + j 0.0 prefault per unit voltage

$$I_1 = \frac{E_a}{Z_1 + Z_2} = \frac{1.0 + j0.0}{j0.25 + j0.35}$$

i.e. $I_1 = -j1.667$

$I_2 = +j1.667$

$I_0 = 0$



(no path for zero sequence current)

To find the out the fault current, $I_b = -I_c$

we use the following relations :

$$I_b = I_{b_1} + I_{b_2} + I_{b_0} = I_{b_1} + I_{b_2} = \lambda^2 I_{a_1} + \lambda I_{a_2}$$

$$= (-\frac{1}{2} - j\frac{\sqrt{3}}{2})(-j1.667) + (-\frac{1}{2} + j\frac{\sqrt{3}}{2})(j1.667)$$

$$= j0.833 - 1.4436 - j0.833 - 1.4436$$

$$= -2.8872pu$$

Now base current $= \frac{2500}{\sqrt{3} \times 132} = 1093A$

$\therefore$ Fault. current $= 1093 \times 2.8872 = 3155.71$ A

To find out line - to - line voltage we find out the sequence components of voltages :

$V_1 = E - I_1 Z_1 = 1 + j0.0 - (-j1.667)$

$= 1 - 0.4167 = 0.5833pu$

similary

$V_2 = -I_2 Z_2 = (-j1.667)(j0.35) = 0.5834pu$

i.e. $V_1 = V_2 = 0.5833pu$

$V_a = V_1 + V_2 + V_0 = 0.5833 + 0.5833 + 0 = 1.1666pu$

$V_b = \lambda^2 V_1 + \lambda V_2 + V_0 = (-0.5 - j0866)(0.5833) + (-0.5 + j0.866)(0.566)$

$= -0.5833pu$

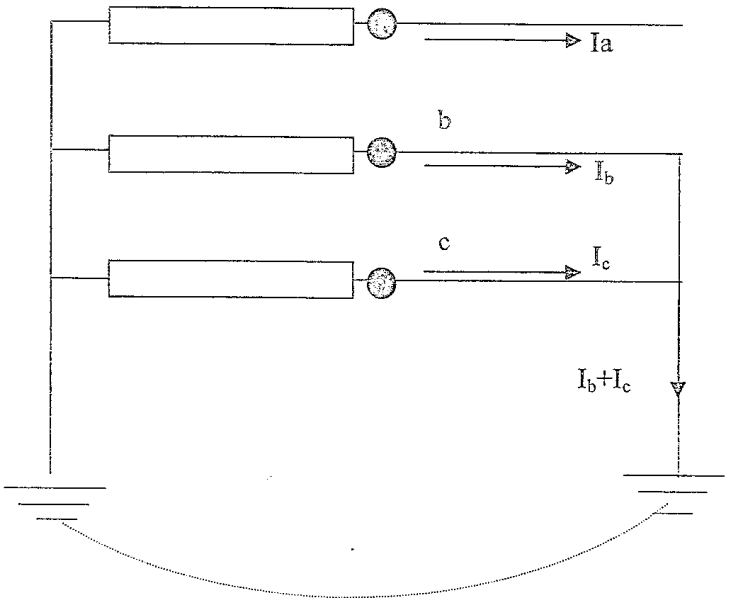
$V_b = -V_c = 0.5833pu$

line. voltage $= V_a - V_b = 1.1666 - (-0.5833) = 1.7499pu$

line to line voltage

$$V_{ab} = 1.7499 \times \frac{13.2}{\sqrt{3}} = 13.33kv$$

Line - to line to Earth fault :



The boundary condition are

$I_a = 0$

$V_b = 0$

$V_c = 0$

$$V_0 = \frac{1}{3}(V_a + V_b + V_c) = \frac{V_a}{3}$$

$$V_1 = \frac{1}{3}(V_a + \lambda V_b + \lambda V_c) = \frac{V_a}{3}$$

$$V_2 = \frac{1}{3}(V_a + \lambda^2 V_b + \lambda V_c) = \frac{V_a}{3}$$

$$\therefore V_1 = V_2 = V_0 = \frac{V_a}{3}$$

again $V_0 = V_1$

in volt drop

$$-I_0Z_0 = E - I_1Z_1 = -I_2Z_2$$

$$I_1Z_1 = E + I_0Z_0$$

$$I_1 = \frac{E + I_0Z_0}{Z_1}$$

also $I_2 = I_0\frac{Z_0}{Z_2}$

But

$$I_a = I_0 + I_1 + I_2 = 0$$

$$= I_0 + \frac{E + I_0Z_0}{Z_1} + I_0\frac{Z_0}{Z_2} = 0$$

$$I_0 + \frac{E + I_0Z_0}{Z_1} + I_0\frac{Z_0}{Z_2} = 0$$

$$I_0 + \frac{E}{Z_1} + I_0\frac{Z_0}{Z_1} + I_0\frac{Z_0}{Z_2} = 0$$

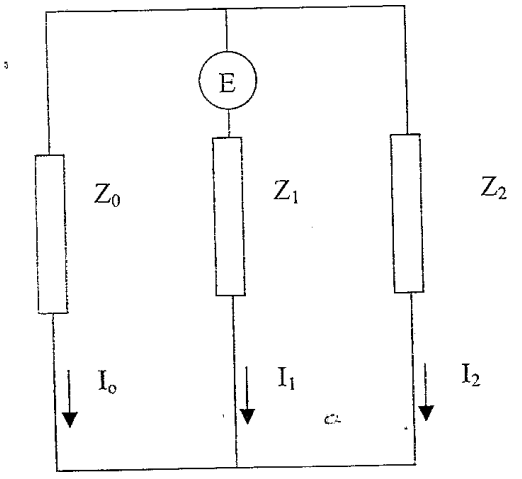
$$(1 + \frac{Z_0}{Z_1} + \frac{Z_0}{Z_2})I_0 = -\frac{E}{Z_1}$$

$$(\frac{Z_1Z_2 + Z_2Z_0 + Z_1Z_0}{Z_2})I_0 = E$$

$$\therefore I_0 = \frac{Z_2E}{Z_0Z_1 + Z_0Z_2 + Z_1Z_2}$$

$$I_f = \frac{3E}{Z_0Z_1 + Z_0Z_2 + Z_1Z_2}$$

the sequence network interconnection is shown in fig (13)



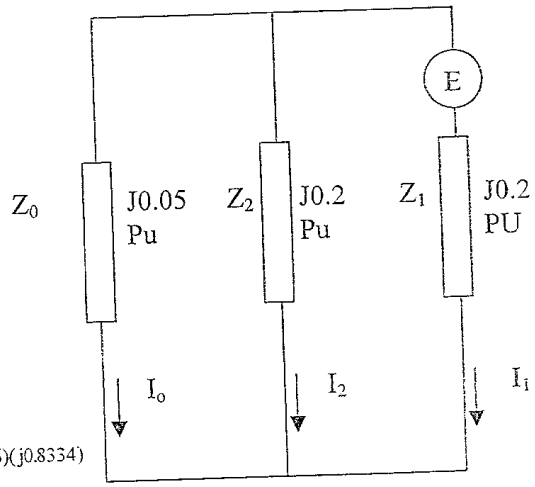
Fig(13)
Interconnection of
Sequence network for
L-L-G fault

Example (5)

A 30 MVA . 11 kv generator has $Z_1 = Z_2 = j0.2pu$ $Z_0 = j0.05pu$, A double line to ground fault occurs on the generator terminal . Determine the line currents , fault current and line to neutral voltages under fault conditions .

Solution :-

fig (14) shown the sequence networks connected for double line to ground fault from fig (14) .



$$I_1 = \frac{1.0\angle 0}{j0.2 + \frac{j0.2 \times j0.05}{j0.25}} = -j4.167pu$$

$$I_2 = -(-j4.167) \times \frac{j0.2}{j0.2 + j0.05} = j3.336pu$$

line current

$I_a = 0$

$I_b = I_0 + \lambda^2 I_1 + \lambda I_2$ substitute for

$I_0 . I_1 , and , I_2$

$I_b = j3.336 + (-0.5 - j0.866)(-j4.167) + (-0.5 + j0.866)(j0.8334)$

$I_b = 10408.1\angle 130.9^\circ$ Amp

and. $I_c = I_0 + \lambda I_1 + \lambda^2 I_2 = j3.336 + (-0.5 + j0.866)(-j4.167) + (-0.5 - j0.866)(j0.8334)$

$I_c = 10408.1\angle 49.1^\circ$ Amp

bearing in mind that I_b , and , I_c are calculated in pu and multiplied by the base current

$$= \frac{3000kvA}{\sqrt{3} \times 11kv} = 1574.6Amp's$$

$V_1 = E - I_1 Z_1$

$V_1 = V_2 = V_0 = 1 - (-j4.167)9j0.2) = 0.1666pu$

$V_a = V_1 + V_2 + V_2 = 0.4998pu$

$V_b = 0$

$V_c = 0$

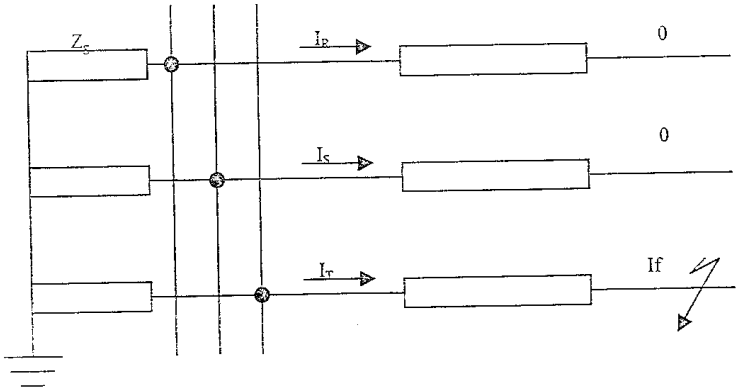
base voltage $= \frac{11}{\sqrt{3}}$

$\therefore$ actual value of $V_a = 0.4998 \times \frac{11}{\sqrt{3}}$

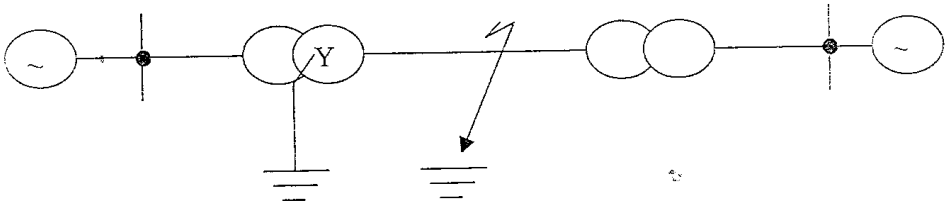
$= 3.174kv$

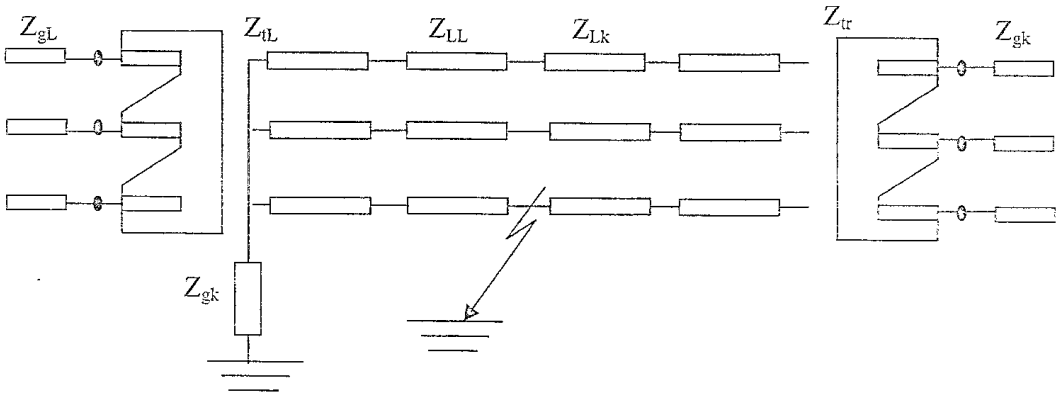
Residual Currents

The currents which flow in the healthy phases during a single phase-to-earth fault if a line a line is fed from one end only , as shown below the residual currents are zeros



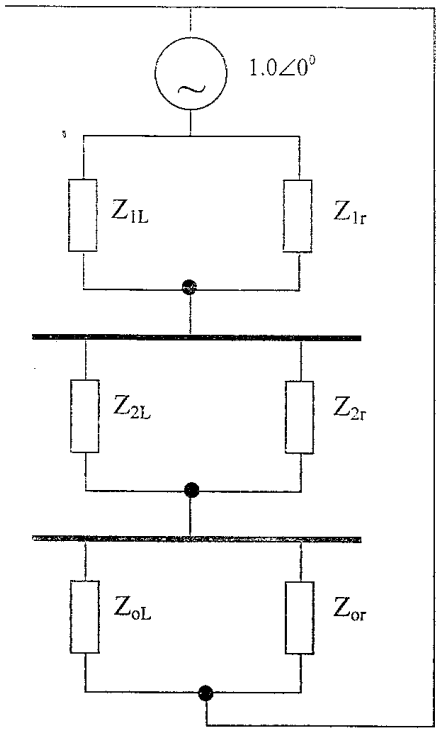
But if the line is fed from both ends as the figure below





Or in 3 phase circuit representation:

The residual currents may flow , depending on the groumoling of the neutral points in the net work . The line to be considered , can be split up in a left section and a right section designated as "L" and "r" .
The sequeuce impedauces connections are as discussed in case of single phase-to-ground fault except that here there are two sources who are feeding the fault in parallel .



Z_{il} = tolal PPS in pedauce to the left of the Bault
$= Z_{igl} + Z_{itrl} + Z_{ile}$
Z_{ir} =total Z_1 on the right of paulf
$Z_{1r} = Z_{1gr} + Z_{1tr} + Z_{ilr}$.
Z_{2l} =Total Nps impedance on the left of the fault .
$Z_{2l} = Z_{2gl} + Z_{2tl} + Z_{2ll}$
Z_{2r} = Total NPS impedance to right of the fault .
$Z_{2r} = Z_{2gr} + Z_{2tr} + Z_{2lr}$
Z_{or} = total ZP3 impedance on the right of fault .
$Z_{ol} = 3Z_e + Z_{oti} + Z_{oll}$
Note : Z_e is multiplied by 3 because it is in the neutral canductor . And Zgo does not appear in this case because neutral is isolated .
$Z_{or} = O + Z_{otr} + Z_{olr}$

$$Z_1 = \frac{Z_{1l}.Z_{1r}}{Z_{1r} + Z_{1r}}$$

$$Z_0 = \frac{Z_{0l}.Z_{0r}}{Z_{0r} + Z_{0r}}$$

$$Z_2 = \frac{Z_{2l}.Z_{2r}}{Z_{2l} + Z_{2r}}$$

$$I_1 = I_2 = I_0 = \frac{E}{Z_1 + Z_2 + Z_0}$$

Those currents are distributed over the left and right sections in accordance with the impedance of each branch .

$$I_{1l} = I_1.\frac{Z_1}{Z_{1l}}$$

$$I_{2l} = I_2.\frac{Z_2}{Z_{2l}}$$

$$I_{0l} = I_0\frac{Z_0}{Z_{0l}}$$

$$I_{1r} = I_1\frac{Z_1}{Z_{1r}}$$

$$I_{2r} = I_2.\frac{Z_2}{Z_{2r}}$$

$$I_{0r} = I_0.\frac{Z_0}{Z_{0r}}$$

The residual current in phoses in the left section .

$I_{sl} = I_{1L} + I_{2L} + I_{0L}$

When $Z_1 = Z_2$ then $I_1 = I_2$

$I_{S1} = I_{1e}$ ( ) $+ I_{0l}$

out of which follows : $I_{sl} = I_{0l} - I_{1l}$ .....(1)

Similarly in phase T : $I_{TL} + I_{1L} + I_{2L} + I_{0L}$

But $I_1 = I_2$ $I_{TL} = I_{0L} - I_{1L}$ .....(2)

In the same way the residual currents , which are supplied from the right , can be obtained .

In phase S $I_{sr} = I_{0r} - I_{1r}$ .....(3)

In phase R $I_{rr} = I_{0r} - I_{1r}$ .....(4)

The fault current in phase R

$I_{RL} = I_{1L} + I_{2L} + I_{0L}$

Or with $I_1 = I_2$, $I_{RL} = 2I_{1L} + I_{0L}$ .....(5)

$I_{Rr} = 2I_{1r} + I_{0r}$ .....(6)

The lotal fault current :

$I_e = I_{RL} + I_{Rr}$

Summary

$$I_1 = I_2 = I_0 = \frac{E}{Z_1 + Z_2 + Z_0}$$

$$I_{1l} = I_1 \frac{Z_1}{Z_{1l}}$$

$$I_{1r} = I_1 . \frac{Z_1}{Z_{1r}}$$

$$I_{2l} = I_2 \frac{Z_2}{Z_{2l}}$$

$$I_{2r} = I_2 . \frac{Z_2}{Z_{2r}}$$

$$I_{0l} = I_0 \frac{Z_0}{Z_{0l}}$$

$$I_{0r} = I_0 . \frac{Z_0}{Z_{0r}}$$

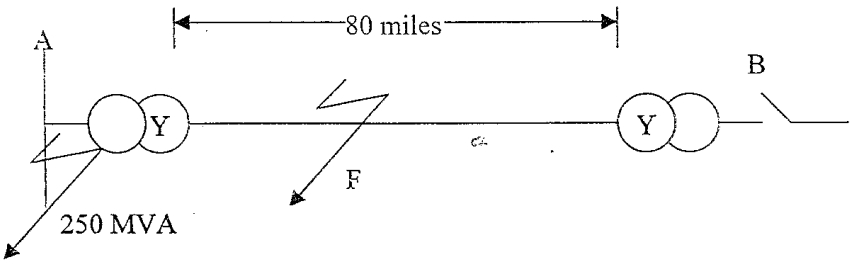
$I_{el} = 3I_{oe}$ $\qquad$ $I_{er} = 3I_{or}$

$I_{Rl} = 2I_{1L} + I_{0L}$ $\qquad$ $I_{Rr} = 2I_{1r} + I_{0r}$

Example :

The singie line diagram of fig (14) represents part of a 3 phase supply system . Transforner lomplexor groupings are as indicated and breaker B is open . The 132 lv transmission line has positive , negative and Zero-phase-sequeuce impedauce of j 0.6 , j 0.6 And j1.2 /mile respectively.Each transformer is rated at 50 MVA and has a reactance of 0.1 pu and the short circuif level at the busbar A is 2500 MVA .

(a) If earth fault occurs on one phase at a point midway along the transmission line , find the value in amps of the fault current and the currents in the two sound phases (residual current) .



Fig(14) Short circuit level

Solution :

(a) Assuming a 100 MVA base , a source short circuit level of 2500 MVA may be represented by a pu source of P.P.S in pedance .

x_{s1} = 0.04 pu , and of n.p.s in pedance , x_{s2} = 0.04 pu (line infeed) each transformer is rated 0.1 pu.at 50 MVA which means 0.2 at 100 MVA .

Transformer reactawce $x_{r1} = x_{r2} = x_{r0}$ =o.2 pu

$$X_{p.u.}\,permile = \frac{X\Omega X MVArating}{(kv)^2}$$

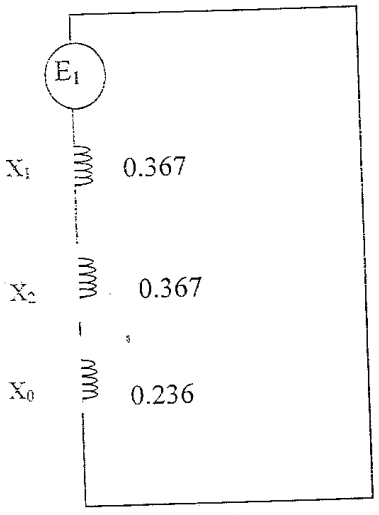
$$X_{L1} = X_{L2}\,permile = \frac{0.6\times 100}{(132)^2} = 0.0034\,pu$$

And by proportion x_{L0} per mile = 0.0068 pu

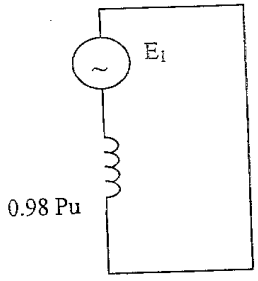
For a 40 mile line

$X_{L1} = X_{L2}$ = 0.136 pu and X_{L0} =0.272 pu

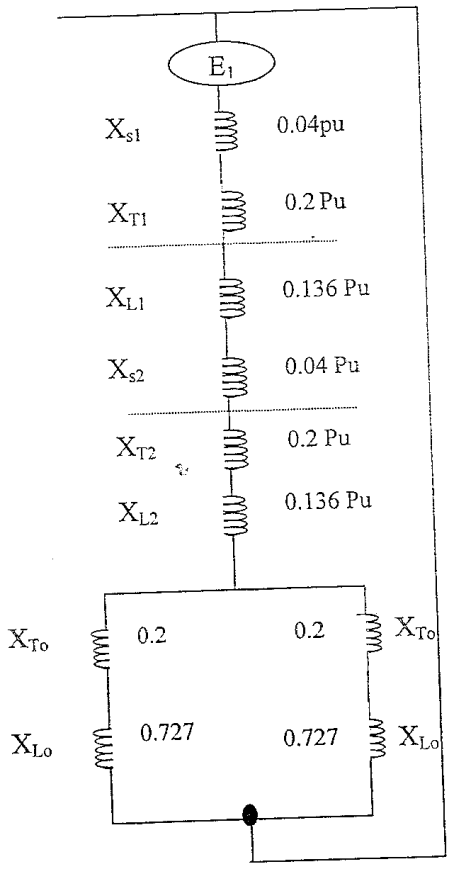
it is now possible to draw the seguence network as shown in fig (15) .



Fig(15)b



Fig(15)c



Fig(15) a

Note: It is clear that there is an equal sharing of ZPS currents I_0' and I_0'' the two parallel brauch as of the ZPS network . Not how the star / delta transformer , from a zero sequence current trap .

The fault current is given by 3 I_o :

$$I_F = 3I_o = 3x\frac{1}{0.98} = 3.1pu$$

$$base current = \frac{100x10^3}{\sqrt{3}x132}$$

$$\therefore actual Fault current = 3.1x\frac{100x10^3}{\sqrt{3}x132}$$

$$= 1360A$$

$$ZPS current, I_o = \frac{1360}{3} = 453.3A$$

Equal division of I_o' and I_o'' gives sound phase currents each equal to :

$$I_o = \frac{453.3}{2} = 22667A$$

SHORT CIRCUIT CALCULATIONS

Part II

FAULT CALCULATION FOR LARGE POWER SYSTEM

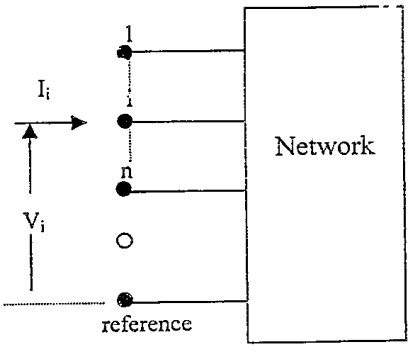
1. INTRODUCTION

The symmetrical and unsymmetrical fault calculations were discussed in part 1 . The network reduction method discussed there can be used only for very small systems . The actual power systems are very large in general . Therefore a systematic procedure suitable for digital computer calculation is necessary . As we have seen earlier , system studies can be carried out using Y_{bus} or Z_{bus} .Y_{bus} is used for load flow computations . For fault calculations (also known as short circuit studies) the use of Z_{bus} is preferable. As compared to the load flow studies , the short circuit studies are simpler because a direct solution is possible and no iterations are required .

However all the three sequences components and sequence networks are involved in the short circuit studies .We can use Y_{bus} whose formation method is straight forword to get Z_{bus} by inversion, but the procedure is laborious. There is another method referred as the step by step procedure of forming Z_{bus} which is of course preferable method.

Bus impedance Matrix:

Fig(1) shows a n bus Passive linear network. The voltage of the i^{th} bus (with respect to reference) Is V_i and the current Entering the i^{th} bus is Ii. From network theory, this network can be described by the following system of equations:



Fig(1)

$$\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_i \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & . & . & Z_{1n} \\ Z_{21} & Z_{22} & . & . & Z_{2n} \\ . & . & . & . & . \\ Z_{i1} & Z_{i2} & . & . & Z_{in} \\ . & . & . & . & . \\ Z_{1n} & Z_{2n} & . & . & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_i \\ . \\ . \\ I_n \end{bmatrix} \quad \text{.....(1)}$$

Or simply $V_{bus} = Z_{bus} I_{bus}$

The general entry Z_{ik} of Z_{bus} can be obtained from the equation:

$$Z_{ik} = \frac{V_i}{I_k}\Big|_{I_1=I_2=I_3 \ldots\ldots = I_n = 0} \quad \text{.....(3)}$$

$$I_k \neq 0$$

It is possible to calculate Zik from equation (3). Terminate all busses in open circuit except the K bus which is terminated in a current source of strength Ik. Solve for V_i and calculate Z_{ik}. Specially if the current injected in bus K is 1.0

Then $K = Z_{ik}$. However this method is not Suitable because calculation of Vi is generally very difficult. The following step by step method is very simple and suitable for computer work..

3. Algorithm for Formulating (Zbus)

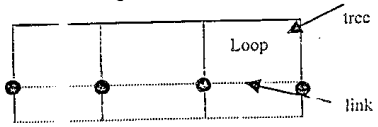
The Algorithm of formulation (Zbus) is described in terms of modifying an existing bus impedance matrix designated as (Zbus) old. The new modified matrix is designated as (Zbus) new. The network consists of a reference bus and a number of other busses. When a new element having self impedance Zs is added, a new bus may be created (if the new element is a tree branch) or a new bus may not be created (if the new element is a link)[1] . Each of these two cases can be subdivided into two cases so that Z_s may be added in the following ways:

---

[1] From Network topology

Tree: whenever branches are connected in such a way as to include all notes yet failing to close a loop- this set of branches is called a tree an the branches in included in this tree is called a tree branch .

Link : any branch is closing a new loop in the tree is called a link .



1. Adding Z_s from a bus to reference
2. Adding Z_s from a new bus to an old bus
3. Adding Z_s from an old bus to reference
4. Adding Z_s between two old busses.

In the following discussion I and k denote old busses and q denoted new bus . The bus impedance matrix is always diagonally symmetric i.e $Z_{ik}=Z_{ki}$.

3-1 Type 1 Modification (Addition of tree branch from a New Bus q to reference) :

fig (2) shows a branch Z_s connected between reference bus and a new bus q. for this new branch we can write :

$V_q = Z_s I_q$ .....(4)

On adding equation (4) to the set of equation (1) we get :

$$\begin{bmatrix} V_1 \\ \cdot \\ V_i \\ \cdot \\ V_n \\ V_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & . & . & . & Z_{1n} & O \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{i1} & Z_{i2} & . & . & . & Z_{in} & O \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & . & . & . & Z_{nn} & O \\ O & O & . & . & . & O & Z_s \end{bmatrix} \begin{bmatrix} I_1 \\ \cdot \\ I_i \\ \cdot \\ I_n \\ I_q \end{bmatrix} \quad \text{.....(5)}$$

$[Z_{bus}]$ old is an nxn matrix . The addition of the new branch means the addition of an (n+1)th row and (n+1)th column . As seen from equation (5) .

$Z_{iq} = Z_{qi} = 0$ for I=1,2,....i.....n

$Z_{qq} = Z_s$

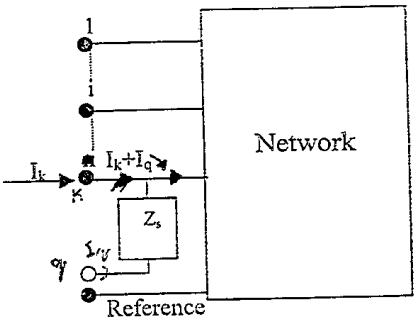
$[Z_{bus}]$ new can , therefore , be written as :

$$[Z_{bus}]_{new} = \begin{bmatrix} & & O \\ & [Z_{bus}]_{old} & \cdot \\ & & O \\ O & . & O Z_s \end{bmatrix} \quad \text{.....(6)}$$

3-2 Type 2 Modification (Addition of a tree branch Zs from a New bus q to old bus k) :

Fig (3) shows this modification .
The equation for the new branch
Is :

$$V_q = Z_s I_q + V_k$$
$$= Z_s I_q + Z_{k1} I_1 + Z_{k2} I_2 + \ldots + Z_{kk} (I_k + I_q) + \ldots + Z_n I_n$$
$$= Z_{k1} I_1 + Z_{k2} I_2 + \ldots + Z_{kk} I_k + \ldots + Z_{kn} I_n + (Z_{kk} + Z_s) I_q \quad \ldots\ldots(7)$$



Fig(3) type 2 modification

The equation for V_1 (i.e first equation (1)) can be written as under keeping in mind that new current at k^{th} bus is $I_k + I_q$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1k} (I_k + I_q) + \ldots + Z_{1n} I_q$$
$$= Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1k} I_k + \ldots + Z_{1n} I_n + Z_{1k} I_q$$

The equation for V_2 ,V_3 .... V_n also get modified in a similar manner . When Equation (7) is added to the modified equations for V_1 , V_2 ,...,V_n . The complete new set of equations is given by :

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_k \\ \cdot \\ \cdot \\ V_n \\ V_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdot & \cdot & Z_{1k} & \cdot & \cdot & Z_{1n} & Z_{1k} \\ Z_{21} & Z_{22} & \cdot & \cdot & Z_{2k} & \cdot & \cdot & Z_{2n} & Z_{2k} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{k1} & Z_{k2} & \cdot & \cdot & Z_{kk} & \cdot & \cdot & Z_{kn} & Z_{kk} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{n1} & Z_{n2} & \cdot & \cdot & Z_{nk} & \cdot & \cdot & Z_{nn} & Z_{nk} \\ Z_{k1} & Z_{k2} & \cdot & \cdot & Z_{kk} & \cdot & \cdot & Z_{kn} & (Z_{nn}+Z_s) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ I_k \\ I_n \\ I_q \end{bmatrix}$$

3-3 Type 3 Modifications (addition of a link Zs between an old Bus k and Reference)

Reference to fig (4) . This case can treated as an extension of type (2) . modification initially let Z_s connected to between a new bus q and old bus k and then let the new bus q be connected so that $V_q = 0$ the new set of equation is :

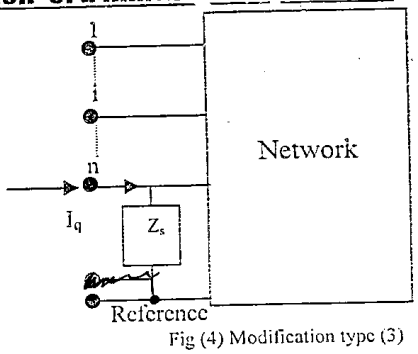


Fig (4) Modification type (3)

$$\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \\ O \end{bmatrix} \begin{bmatrix} [Z_{bus}]_{old} & & Z_{1k} \\ & & Z_{2k} \\ & & . \\ & & . \\ & & Z_{nk} \\ Z_{k1} & Z_{k2} \ . \ . \ . \ Z_{kn} & Z_{kk}+Z_s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ .. \\ I_n \\ I_q \end{bmatrix} \quad \text{.....(11)}$$

If I_q is eliminated from equation (11) i.e last row and last column of the above impedance matrix are eliminated (using matrix algebra) we have

$$[Z_{bus}]_{new} = [Z_{bus}]_{old} - \frac{1}{Z_{kk}+Z_s} \begin{bmatrix} Z_{1k} \\ Z_{2k} \\ . \\ . \\ Z_{nk} \end{bmatrix} [Z_{k1} \quad Z_{k2} \quad . \quad . \quad . \quad Z_{kn}] \text{.....(12)}$$

3.4 Type 4 Modification [Addition of link Zs between two old busses] :

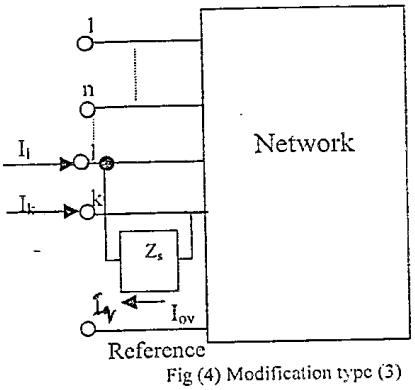


Fig (4) Modification type (3)

Refferring to fig (5) . Equation for V_1 can be written as :

$V_1 = Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1l} (I_l + I_q) + Z_{1j} I_j + Z_{1k} (I_k - I_q) + \ldots + Z_{1n} I_n$ ....(13)

Rearranging

$V_1 = Z_{11} I_1 + Z_{12} I_2 + \ldots + Z_{1n} I_n + (Z_{1l} - Z_{1k}) I_q$ ...(13b)

Equation similar to equation (13b) can be written for all busses . It is to be noted that :

$V_k = Z_s I_q + V_1$ ..... (14)

Or $Z_{k1} I_1 + Z_{k2} I_2 + \ldots + Z_{ki} (I_l + I_q) + Z_{kk} (I_k - I_q) + \ldots = Z_s I_q + Z_{i1} I_1 + Z_{i2} I_2 + \ldots + Z_{ii} (I_l + I_q) + Z_{ij} I_j + Z_{ik} (I_k - I_q)$..... (14b)

Rearranging

$O = (Z_{i1} - Z_{k1}) I_1 + \ldots + (Z_{ii} - Z_{ki}) I_l + (Z_{ij} - Z_{kj}) I_j + (Z_{ik} - Z_{kk}) I_k + \ldots + (Z_s + Z_{ii} - Z_{ki} + Z_{kk}) I_q$ .....(14c)

Equation (13b) along with similar equations for V_2 , V_3,.....V_n and Equation (14c) can be written in the following form .

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_n \\ o \end{bmatrix} = \begin{bmatrix} & Z_{1i}-Z_{1k} & \\ & Z_{2i}-Z_{2k} & \\ [Z_{bus}]_{old} & \cdot & \\ & \cdot & \\ & \cdot & \\ (Z_{i1}-Z_{k1}) & \cdots & Z_{ii}+Z_{kk}-2Z_{ik} \end{bmatrix} \begin{bmatrix} Z_{1i}-Z_{1k} \\ Z_{2i}-Z_{2k} \\ \vdots \\ \vdots \\ I_n \\ \vdots \\ I_q \end{bmatrix} \quad \text{---(15)}$$

Eliminating I_q from Equation (15) (by matrix Algebra : The method is given in end) we get

$$[Z_{bus}]_{new} = [Z_{bus}]_{old} - \frac{1}{Z_s + Z_{ii} + Z_{kk} - 2Z_{ik}} x \begin{bmatrix} Z_{1i}-Z_{1k} \\ Z_{2i}-Z_{2k} \\ \cdot \\ \cdot \\ Z_{ni}-Z_{nk} \end{bmatrix} x[(Z_{i1}-Z_{k1}).....(Z_{in}-Z_{kn})].....(16)$$

3.5 Modification of [Zbus] for Changes in Network

When a change is made in a network , it is not necessary to frame the new bus impedance matrix from ab-initio . The original matrix can be modified to give the new matrix .

The methods discussed above can be used if new elements are added in the network . Any number of new elements (branches or links) can be added one at a time . If any element is to be removed , the new matrix can be obtained by adding in parallel with the element a link whose impedance is equal to negative of the impedance of the element to be removed . If the impedance of any element is to be changed , the new matrix can be obtained by adding a link in parallel with that element such that their equivalent impedance is equal to the desired value . The method discussed in appendix B can be used to eliminate a bus .

Example

Fig . (6) shows a 4 bus system . Treating bus 4 as the reference bus obtain $[Z_{bus}]$.

Solution

1. We start with the 1 ohm element between busses 1 and 4 .

Type 1 modification gives .

$[Z_{bus}] = [1]$

2. Add the 1 ohm element between busses 2 and 4 . Type 1 modification gives .

$$[Z_{bus}]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

3.Now add the 1 ohm element between busses 3 and 4 .
Type 1 modification gives .

$$[Z_{bus}]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

4.Next add the 1 ohm element between the old busses 1 and 2 .
Type 4 modification gives .

$$[Z_{bus}]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\frac{1}{1+1+1}\begin{bmatrix}1\\ -1\\ 0\end{bmatrix}\begin{bmatrix}1 & -1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}-\frac{1}{3}\begin{bmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{2}{3} & 0\\ 0 & 0 & 1\end{bmatrix}$$

5.Finally add the 1 ohm element between the old busses 1 and 3 .
Type 4 modification gives .

$$[Z_{bus}]=\begin{bmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{2}{3} & 0\\ 0 & 0 & 1\end{bmatrix}-\frac{1}{1+\frac{2}{3}+1}\begin{bmatrix}\frac{2}{3}\\ \frac{1}{3}\\ -1\end{bmatrix}\begin{bmatrix}\frac{2}{3} & \frac{1}{3} & -1\end{bmatrix}=\begin{bmatrix}\frac{2}{3} & \frac{1}{3} & 0\\ \frac{1}{3} & \frac{2}{3} & 0\\ 0 & 0 & 1\end{bmatrix}-\frac{3}{8}\begin{bmatrix}\frac{4}{9} & \frac{2}{9} & \frac{-2}{3}\\ \frac{2}{9} & \frac{1}{9} & \frac{-1}{3}\\ \frac{-2}{3} & \frac{-1}{3} & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & \frac{1}{4} & \frac{1}{4}\\ \frac{1}{4} & \frac{5}{8} & \frac{1}{8}\\ \frac{1}{4} & \frac{1}{8} & \frac{5}{8}\end{bmatrix}$$

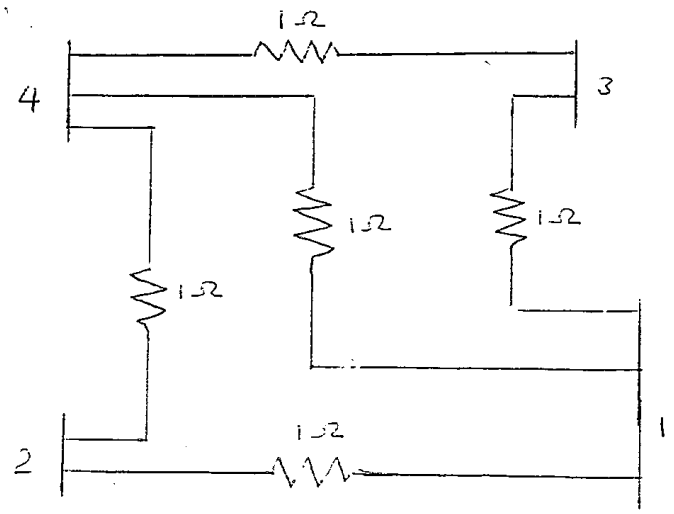


Fig. (6)

14.4 SEQUENCE MATRICES

To start the discussion of short circuit studies, we define the following notations :

V_{0-bus}=zero sequence bus voltage vector ($n \times 1$), general entry V_k^0

V_{1-bus}=positive sequence bus voltage vector ($n \times 1$), general entry V_k^1

V_{2-bus}=negative sequence bus voltage vector ($n \times 1$), general entry V_k^2

I_{0-bus}=zero sequence bus current vector ($n \times 1$), general entry I_k^0

I_{1-bus}=positive sequence bus current vector ($n \times 1$), general entry I_k^1

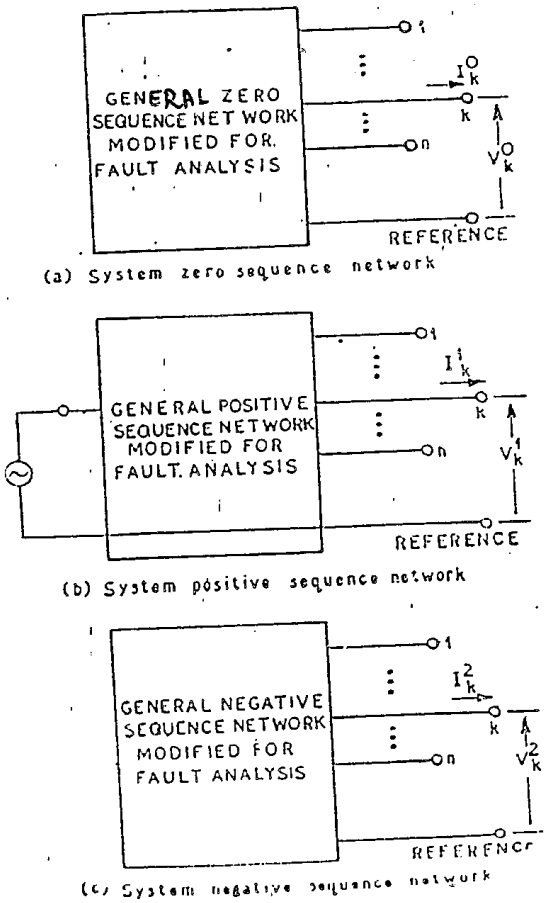
I_{2-bus}=negative sequence bus current vector ($n \times 1$), general entry I_k^2

In the symbol V_k^0, I_k^2, etc., the subscript refers to the bus number and the super script indicates the sequence as shown in Fig. 7. All these sequence components are for phase a of the network. The subscript a has been dropped for convenience in writing. Thus V_{0-bus} means V_{a0-bus} and so on. It is seen that the direction of current in Fig. 7 is opposite to that shown in Fig. 1. However the direction shown in Fig. 7 is more natural (because fault currents, in general, flow out of the busses) and agrees with that used in chapter-10. The sequence impedance matrices required for the short circuit studies are :

$[Z_{0-bus}]$=zero sequence bus impedance matrix ($n \times n$), general entry Z^0_{ik}

$[Z_{1-bus}]$=positive sequence bus impedance matrix ($n \times n$), general entry Z^1_{ik}

$[Z_{2-bus}]$=negative sequence bus impedance matrix ($n \times n$), general entry Z^2_{ik}


GENERAL ZERO SEQUENCE NET WORK MODIFIED FOR FAULT ANALYSIS
1
k
n
REFERENCE
GENERAL POSITIVE SEQUENCE NETWORK MODIFIED FOR FAULT. ANALYSIS
GENERAL NEGATIVE SEQUENCE NETWORK MODIFIED FOR FAULT ANALYSIS

(a) System zero sequence network

(b) System positive sequence network

(c) System negative sequence network

Fig. (7) Sequence Networks Modified for Fault Analysis

The sequence impedance matrices can be formulated by the method discussed in section .3. All these matrices refer to the passive sequence network. The positive sequence impedance network contains active sources. While forming the positive sequence bus impedance matrix the emfs of sources are assumed to be short circuited.

A somewhat simplified, although approximate, short circuit study can be made by neglecting the prefault currents. This means that all the bus voltages are 1 *pu* immediately before the fault. The equations relating the sequence quantities are :

$$V_{0\text{-}bus} = -[Z_{0\text{-}bus}]\, I_{0\text{-}bus} \qquad 17\text{ a)}$$

$$V_{1\text{-}bus} = E_{bus} - [Z_{1\text{-}bus}]\, I_{1\text{-}bus} \qquad 17\text{ b)}$$

$$V_{2\text{-}bus} = -[Z_{2\text{-}bus}]\, I_{2\text{-}bus} \qquad 17\text{ c)}$$

Note the negative sign in the above equations (whereas Eq. .1 does not have a negative sign). This is due to the fact that the direction of current in Fig. .7 is opposite to that in Fig. 1. Since the prefault currents are neglected, vector E contains $1\angle 0$ in all entries. The currents are all zero till the network is terminated externally. At a time only one bus (i.e., the faulted bus k) is terminated. Thus only I^0_k, I^1_k, I^2_k have non-zero entry. Very frequently $[Z_{1\text{-}bus}]$ and $[Z_{2\text{-}bus}]$ are assumed to be identical to reduce computer memory requirement.

3.5 EQUATIONS FOR SHORT CIRCUIT STUDIES

The equations for short circuit studies are developed by using Eqs. (.17) and terminating the network at the faulted bus (bus k).

5.1 Symmetrical Fault

For a symmetrical fault the negative sequence and zero sequence quantities are absent, i. e., $V_{0\text{-}bus}$, $V_{2\text{-}bus}$, $I_{0\text{-}bus}$ and $I_{2\text{-}bus}$ are zero.

$$V^1_k = E - (Z^1_{k1}\, I^1_1 + Z^1_{k2}\, I^1_2 + \ldots\ldots + Z^1_{kk}\, I^1_k + \ldots\ldots + Z^1_{kn}\, I^1_n) \qquad (.18)$$

But all currents except the current at the faulted bus, i. e., I^1_k are zero Therefore,

$$V^1_k = E - Z^1_{kk}\, I^1_k \qquad (\ 19)$$

If Z_f is the fault impedance

$$V^1_k = I^1_k\, Z_f \qquad (\ 20)$$

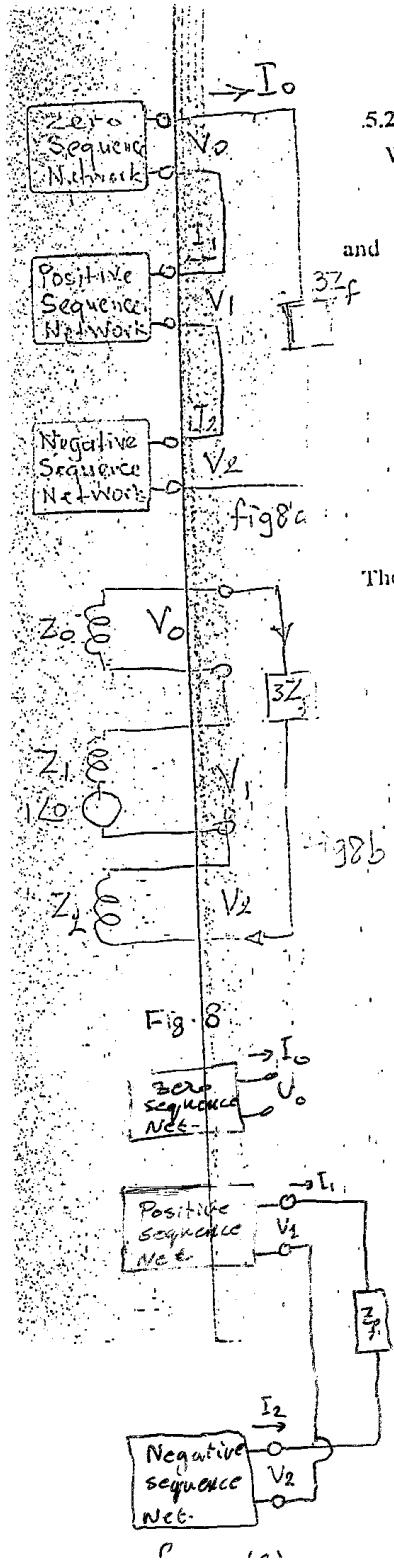
From Eqs. (.19 & .20)

$$I^1_k = \frac{E}{Z^1_{kk} + Z_f} \qquad (\ .21)$$

The voltage at ith bus is

$$V^1_i = E - Z^1_{ik}\, I^1_k = E\left(1 - \frac{Z^1_{ik}}{Z^1_{kk} + Z_f}\right)$$

$$\text{for } i = 1, 2 \ldots\ldots\ldots n \qquad (\ .22)$$



5.2 Single Line to Ground Fault

fig (8)

We know that

$$I^0_i = I^1_i = I^2_i = 0 \text{ for } i = 1, 2 \ldots\ldots n \quad (i \neq k) \qquad (23a)$$

and

$$I^0_k = I^1_k = I^2_k \qquad (23b)$$

$$V^0_k + V^1_k + V^2_k = 3Z_f \; I^1_k \qquad (24)$$

From Eq. (11.17)

$$V^0_k = - Z^0_{kk} \, I^0_k$$

$$V^1_k = E - Z^1_{kk} I^1_k \qquad (25)$$

$$V^2_k = - Z^2_{kk} \, I^2_k$$

Combining Eqs. (23 b, 24 & 25)

$$I^1_k = \frac{E}{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3Z_f} \qquad (26)$$

The sequence components of bus voltages at ith bus ($i = 1, 2 \ldots n$) are

$$V_i^0 = - Z^0_{ik} \, I^0_k = - Z^0_{ik} \, I^1_k \qquad (27a)$$

$$= \frac{-Z_{ik^0} \, E}{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3Z_f} \qquad (27b)$$

$$V_i^1 = E - Z^1_{ik} \, I^1_k \qquad (28a)$$

$$= E \left[1 - \frac{Z^1_{ik}}{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3 \, Z_f} \right] \qquad (28b)$$

$$= E \left[\frac{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3 \, Z_f - Z^1_{ik}}{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3 \, Z_f} \right] \qquad (28c)$$

$$V_i^2 = - Z^2_{ik} \, I^2_k = - Z^2_{ik} \, I^1_k \qquad (29a)$$

$$= \frac{-Z^2_{ik} \, E}{Z^0_{kk} + Z^1_{kk} + Z^2_{kk} + 3Z_f} \qquad (29b)$$

5.3 Line to Line Fault

fig (9)

For a line to line fault

$$V_{0\text{-}bus} = I_{0\text{-}bus} = 0. \qquad (30)$$

Eqs. (23a and 25) are still valid

$$V^1_k = V^2_k + I^1_k Z_f \qquad (31)$$

Substituting the values for V^1_k and V^2_k from Eq. (25) into Eq. (31)

$$E - Z^1_{kk} \, I^1_k = - Z^2_{kk} \, I^2_k + I^1_k \, Z_f \qquad (32)$$

We know that for a line to line fault $I^2_k = -I^1_k$. Making this substitution in Eq. (.32)

$$E - Z^1_{kk}\, I^1_k = Z^2_{kk}\, I^1_k + I^1_k\, Z_f$$

or
$$I^1_k = \frac{E}{Z^1_{kk} + Z^2_{kk} + Z_f} \qquad .33)$$

The positive and negative sequence voltages at ith bus ($i = 1, 2 \ldots\ldots n$) are

$$V^1_i = E - Z^1_{ik}\, I^1_k \qquad (\ .31\,a)$$

$$= E - Z^1_{ik} \frac{E}{Z^1_{kk} + Z^2_{kk} + Z_f} \qquad (\ .34\ b)$$

$$= E\left[\frac{Z^1_{kk} + Z^2_{kk} + Z_f - Z^1_{ik}}{Z^1_{kk} + Z^2_{kk} + Z_f}\right] \qquad (\ .34\ c)$$

$$V^2_i = -Z^2_{ik}\, I^2_k \qquad (\ .35\ a)$$

$$= + Z^2_{ik}\, I^1_k \qquad (\ .35\ b)$$

$$= \frac{Z^2_{ik}\, E}{Z^1_{kk} + Z^2_{kk} + Z_f} \qquad (\ .35\ c)$$

.5.4 Double Line to Ground Fault

From the equivalent circuit shown in Fig. (10.3) it can be inferred that

$$V^1_k = V^2_k \qquad (\ .36\ a)$$

$$V^0_k - V^1_k = 3 I^0_k Z_f \qquad (\ .36\ b)$$

and
$$I^0_k + I^1_k + I^2_k = 0 \qquad (\ .37)$$

Eqs. (.23a and . .25) are still valid. From Eqs. (.25, .36 and .37)

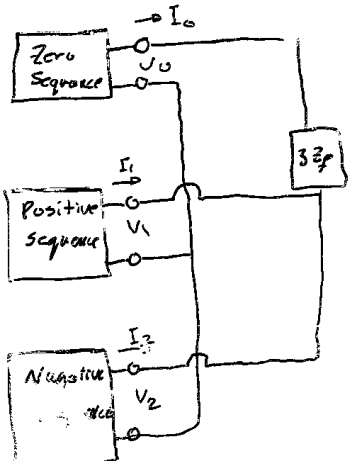
$$I^1_k = \frac{E - V^1_k}{Z^1_{kk}} \qquad (\ .38\ a)$$

$$I^2_k = -\frac{V^2_k}{Z^2_{kk}} = -\frac{V^1_k}{Z^2_{kk}} \qquad (\ .38\ b)$$

$$I^0_k = -\frac{V^0_k}{Z^0_{kk}} = \frac{-V^1_k}{Z^0_{kk} + 3 Z_f} \qquad (\ .38\ c)$$

Substituting Eqs. (.38) into Eq. (.37) we have

$$-\frac{V^1_k}{Z^0_{kk} + 3 Z_f} + \frac{E - V^1_k}{Z^1_{kk}} - \frac{V^1_k}{Z^2_{kk}} = 0$$



WHY WE GROUND AN AUTOTRANSFORMER

Power autotransformers are quite frequently used in public-utility power transmission systems. These systems may be subjected to dangerous fundamental frequency overvoltage during system faults. Since the magnitude of this overvoltage depends in part upon the method of grounding the system and autotransformer, the nature of this overvoltage will be explained.

Consider the case of ungrounded system using an autotransformer as represented in Fig 1.

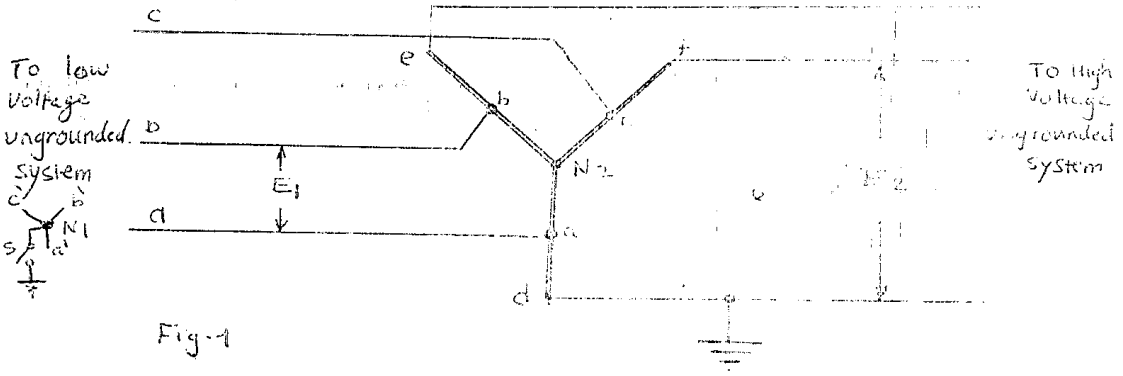


Fig-1

Lines a, b, c represent the low-voltage system normally operating at line-to-line voltage E_1, and points d, e, f represent the terminals of a step-up autotransformer normally operating at line to line voltage E_2. In the event of a line-to-ground fault on the line connected to terminal d, the low-voltage phases b and c are elevated above-ground by the amount

$$E_{db} = E_{dc} = \frac{1}{\sqrt{3}} \sqrt{E_1^2 + E_2^2 + E_1 E_2}$$

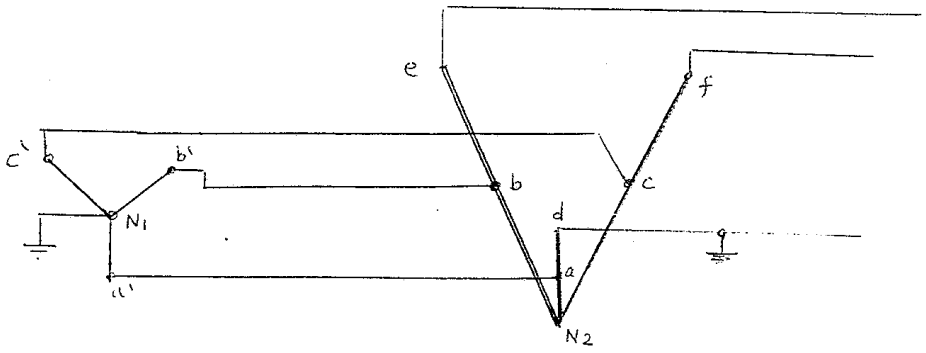
For example, in the case of an autotransformer rated 132/400 KV operating ungrounded system, a line-to-ground fault on one of the high-voltage lines will impress a voltage to ground on two of the low-voltage lines of

$$0.58\sqrt{(132)^2+(400)^2+(132)(400)} = 278.29 \text{ Volts}$$

obviously, this is an undesirable situation and cannot be tolerated.

Solid grounding of the autotransformer neutral eliminates this type of overvoltage.

Consider now the low-voltage system is grounded by closing swich "S". A line-to-ground fault on terminal d forces the voltage of point d to that of N1. This inverts the phase voltage of winding da by impressing voltage N_1a' from point d to a. The da portion of the winding induces in a N_2 portion of the winding a voltage of corresponding phase and of a magnitude depending upon the turn ratio of the two parts of the winding. This results in a shift of poin N_2 as shown in Fig. 2



SYSTEMATIC SHORT-CIRCUT COMPUTATION USING ADMITTANCE MATRIX FORMULATION

In general faulted system studies constitute an integral part of multi-machine transient stability and load flow programs. Usually these programs using Y-matrix representation of the system under study because it is easier to obtain. A preliminary stage to the study is the collection of appropriate data specifying the system analysed in terms of prefault voltage, loading and generating conditions. Such data is then processed to form a nodal equivalent network constitued by admittances and injected current. We shall presently develop a technique that possesses a generality and directly applicable to an n-bus system using digital computer for short circuit computation.

To form the admittance matrix, the following notes must be followed:

1- Transformer, lines and loads are represented by their appropriate admittances as discussed previously.

2- The generators are represented by a constant voltage (E) behined an approximate machine admittance Y_M, the value of which depends on the time of calculation from the instant of fault inception. This is illustrated in Fig. 1(a).

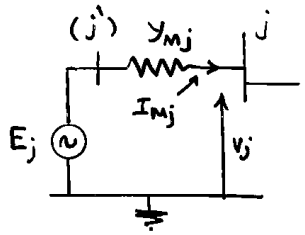


Fig. 1(a)

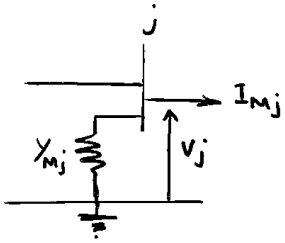


Fig. 1(b)

Notes:

* when analysing the first two or three cycles following the fault, the subtransient admittance of the machine is normally used.

** For longer times, it is more appropriate to use the transient admittence.

The machine model shown in Fig. 1(a) is the converted to a nodal equivalent by means of Norten's Theorem which changes the voltage source into current source injected at the bus j as shown in Fig. 1(b). This is most effective as if we take another node (j') for example.

The injected nodal current is given by

$$I_j = Y_{Mj} E_j \tag{1}$$

where $$E_j = V_j + \frac{I_{Mj}}{Y_{Mj}} \tag{2}$$

$$\text{(3)}$$

so that $$I_j = Y_{Mj} V_j + I_{Mj}$$

I_{Mj} is the current required at the voltage V_j to produce the machine power $P_j + jQ_j$, so

$$(I_{Mj})^* V_j = P_j + jQ_j \tag{4}$$

Thus from the load-flow data of P, Q and V we may calculate the injected nodal current I_j as

$$I_j = Y_{Mj} V_j + \frac{P_j - jQ_j}{V_j^*} \tag{5}$$

Let us take as a reference the small system of Fig. 2(a):

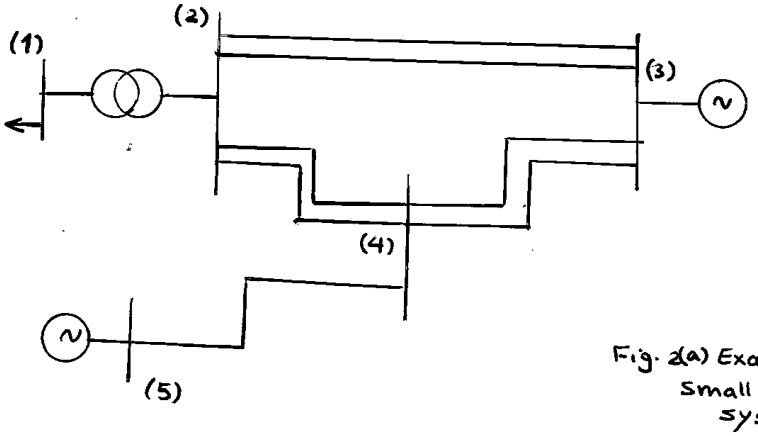


Fig. 2(a) Example for small power system

Each element is converted to its nodal equivalent as shown in Fig 2(b), and finally simplified to the equivalent circuit as shown in Fig 2(c).

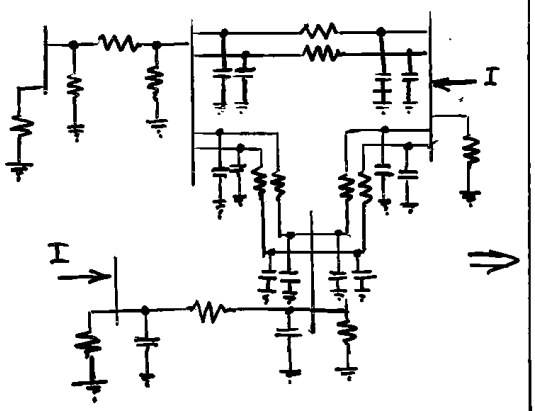


Fig. 2(b)

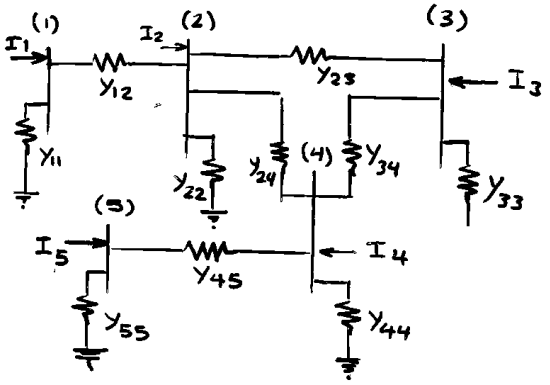


Fig 2(c)

The following equations may be then written for the network:

$$I_1 = Y_{11} V_1 + Y_{12} (V_1 - V_2) \tag{6}$$

$$I_2 = Y_{12} (V_2 - V_1) + Y_{22} V_2 + Y_{23} (V_2 - V_3) + Y_{24} (V_2 - V_4) \tag{7}$$

$$I_3 = Y_{23} (V_3 - V_2) + Y_{33} V_3 + Y_{34} (V_3 - V_4) \tag{8}$$

$$I_4 = Y_{24} (V_4 - V_2) + Y_{34} (V_4 - V_3) + Y_{44} V_4 + Y_{45} (V_4 - V_5) \tag{9}$$

$$I_5 = Y_{45} (V_5 - V_4) + Y_{55} V_5$$

or in matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{21} & Y_{31} & Y_{41} & Y_{51} \\ Y_{12} & Y_{22} & Y_{32} & Y_{42} & Y_{52} \\ Y_{13} & Y_{23} & Y_{33} & Y_{43} & Y_{53} \\ Y_{14} & Y_{24} & Y_{34} & Y_{44} & Y_{54} \\ Y_{15} & Y_{25} & Y_{35} & Y_{45} & Y_{55} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} \tag{10}$$

where $Y_{ii} = \sum_j Y_{ij}$ $\qquad Y_{ij} = -Y_{ij}$ $\qquad i \neq j$

or we can write eq. 12 in the following form

$$[I] = [Y][V] \tag{11}$$

The line current I_{ij} measured at bus i and defined positive in the direction $i \rightarrow j$, equals

$$I_{ij} = (V_i - V_j) Y_{ij} \tag{12}$$

The nodal admittence equation (10) requires a complete iterative solution for each type of fault and location. Instead eq. 11 can be written as:

$$[V] = [Y]^{-1}[I] \qquad (13)$$
$$= [Z][I]$$

This equation uses the bus nodal ampedance matrix $[Z]$ and gives a direct solution of the fault condition at any node. The impedance matrix is obtained by an efficient inversion technique such as "Matlab" using digital computer when very large system is considered.

Fault calculations

From the initial machine data, the values of $[I]$ are first calculated from eq. (5). using one p.u. voltages. These may now be used to obtain a better estimates of $[V]$, the prefault voltage at every node from eq. 13. If the initial data are supplied from a load flow, this calculation will not make any difference.

The program now has sufficient information to calculate the voltages and currents during a fault.

From Fig. 3 the voltage at the fault bus k is

$$V_k^f = Z^f I^f \qquad (14)$$

where k is the bus to be faulted, Zf is the fault impedance and If is the faulted current.

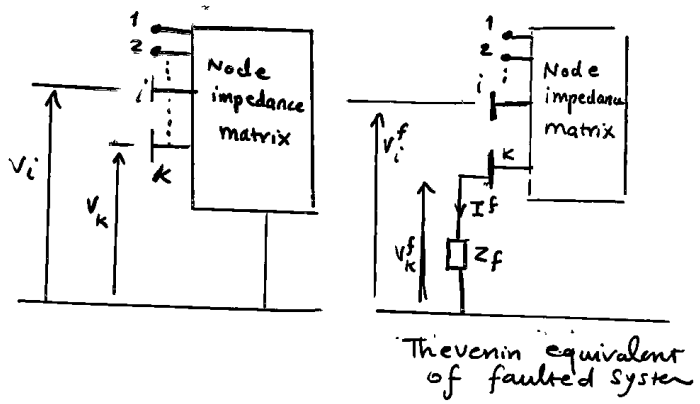


Fig. 3
Thevenin equivalent of prefault system

Thevenin equivalent of faulted system

Eq. (13) may be expanded to yield

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_K \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \cdots Z_{1K} \cdots Z_{1n} \\ Z_{21} & Z_{22} \cdots Z_{2K} \cdots Z_{2n} \\ \cdots & \cdots \\ Z_{K1} & Z_{K2} \cdots Z_{KK} \cdots Z_{Kn} \\ \cdots & \cdots \\ Z_{n1} & Z_{n2} \cdots Z_{nK} \cdots Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_K \\ \vdots \\ I_n \end{bmatrix} \qquad (15)$$

Selecting row K and expanding gives

$$V_K = Z_{K1} I_1 + Z_{K2} I_2 + \cdots + Z_{KK} I_K + \cdots Z_{Kn} I_n . \qquad (16)$$

This equation describe the voltage at bus K prior to the fault. During a fault a large current I^f flows out of bus K.

$$\therefore V_K^f = Z^f I^f = Z_{K1} I_1 + Z_{K2} I_2 + \cdots + (I_K - I^f) Z_{KK} + \cdots + Z_{Kn} I_n$$

$$= Z_{K1} I_1 + Z_{K2} I_2 + \cdots + Z_{KK} I_K + \cdots + Z_{Kn} I_n - I^f Z_{KK} .$$

$$= V_K - Z_{KK} I^f . \qquad (17)$$

$$\therefore Z^f I^f = V_K - Z_{KK} I^f$$

$$\text{or} \quad I^f = \frac{V_K}{Z_{KK} + Z_f} \qquad (18)$$

Also from eq. 15 the prefault voltage at any other bus j is

$$V_j = Z_{j1} I_1 + Z_{j2} I_2 + \cdots + Z_{jK} I_K + \cdots + Z_{jn} I_n$$

and during the fault:

$$V_j^f = Z_{j1} I_1 + Z_{j2} I_2 + \cdots + Z_{jK} I_K + \cdots Z_{jn} I_n - Z_{jK} I_f$$

or

$$V_j^f = V_j - Z_{jK} I^f . \qquad (19)$$

From eq. (18) and (19) the fault voltages at every bus in the system may be calculated. Once I^f is calculated, the nodal new current are calculated as:

$$I_j^f = I_j \qquad \text{for } j \neq K \qquad j = 1 \text{ to } n$$

$$I_K^f = I_K - I^f \qquad \text{for } j = K .$$

For the faulted bus K ; the postfault bus voltage is

$$V_K^f = V_K - Z_{KK} I_f$$

$$= V_K - \frac{Z_{KK}}{Z_{KK}+Z_f} V_K = V_K \left(1 - \frac{Z_{KK}}{Z_{KK}+Z_f}\right)$$

$$\therefore V_K^f = \frac{Z_f}{Z_{KK}+Z_f} V_K \cdot \qquad (20)$$

If the short circuit is solid (that is, $Z_f = 0$), then

$$I_{f_K} = \frac{V_K}{Z_{KK}} \quad \text{or} \quad \left(\frac{V_K}{X_K}\right) \qquad (21)$$

$$V_K^f = 0$$

& $$V_j^f = V_j - \frac{Z_{jK}}{Z_{KK}} V_K \qquad j \neq K$$

(Note that Z_{KK} represents the Thevenin reactance X_{Th}).

The post fault currents in all branches can be obtained from eq. 12 :

$$I_{ij} = (V_i - V_j) Y_{ij}$$

Example: We shall examplify the above general analysis method on a three-bus system shown in Fig. 4. 100 and 200 MVA generators are feeding, via step-up transformers, power into bus 1 and 2 respectively through transmisssion lines of equall impedance, j0.20 p.u. on 100 MVA base. Find the short circuit level at all three buses. All three prefault bus voltages are assumed 1 p.u. Generator plus transformer reactances equal 0.30 p.u based upon respective ratings.

Solution: Fig. 4 represents the generator, transformers and line on 50 MVA base.

We first form the admittance matrix for the network as follows :

$$Y_{11} = \frac{1}{j0.15} + \frac{1}{j0.1} + \frac{1}{j0.1} = -j26.67$$

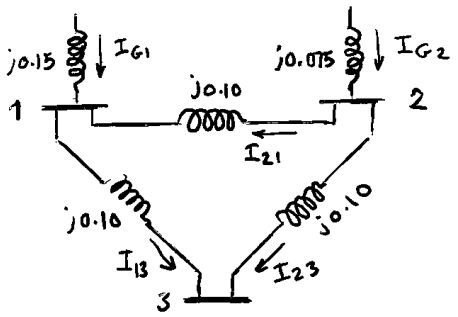
$$Y_{22} = \frac{1}{j0.075} + \frac{1}{j0.1} + \frac{1}{j0.1} = -j33.33$$

$$Y_{33} = \frac{1}{j0.1} + \frac{1}{j0.1} = -j20.0$$

$$Y_{12} = Y_{21} = -\frac{1}{j0.1} = j10$$

$$Y_{13} = Y_{31} = -\frac{1}{j0.1} = j10$$

$$Y_{23} = Y_{32} = -\frac{1}{j0.1} = j10$$



(All values on Base 50MVA)

Fig.4

$$\therefore \quad Y_{bus} = \begin{bmatrix} -26.67 & j10 & j10 \\ j10 & -j33.33 & j10 \\ j10 & j10 & -j20 \end{bmatrix} \quad \& \quad Z_{bus} = Y_{bus}^{-1} = \begin{bmatrix} j0.073 & j0.038 & j0.055 \\ j0.038 & j0.055 & j0.047 \\ j0.055 & j0.047 & j0.1014 \end{bmatrix}$$

let $V_k = 1.0$ p.u. & $Z_f = 0$.

$$\therefore \quad |I^f_{(1)}| = \frac{1}{0.073} = 13.7 \text{ p.u} \qquad \text{for short on bus 1.}$$

$$|I^f_{(2)}| = \frac{1}{0.055} = 17.9 \text{ p.u} \qquad \text{for short on bus 2}$$

$$|I^f_{(3)}| = \frac{1}{0.1014} = 9.86 \text{ p.u} \qquad \text{for short on bus 3}$$

On Base 50MVA, the Fault levels of the three buses are 685, 895 and 493 MVA respectively

For a solid short at bus 3. Eq. 21 revealse that the bus voltages will drop to the following values:

$$V_1^f = 1.0 - \frac{j0.055}{j0.1014} \times 1.0 = 0.45 \text{ p.u.}$$

$$V_2^f = 1.00 - \frac{j0.047}{j0.1014} \times 1.0 = 0.535 \text{ p.u}$$

$$V_3^f = 0.$$

The short circuit current on the three lines are:

$$|I_{12}| = \frac{|0.450 - 0.535|}{0.1} = 0.850 \text{ p.u}$$

$$|I_{23}| = \frac{|0.535 - 0|}{0.1} = 5.35 \text{ p.u.}$$

$$|I_{31}| = \frac{|0 - 0.450|}{0.1} = 4.50 \text{ p.u.}$$

Interpretation of n-bus system

Interpretation of n-bus System

The above three-bus example demonstrates the ease with which the short-circuit computations proceed once the Z_{bus} matrix is known. We found Z_{bus} by first assembling the Y_{bus} matrix and then inverting the latter. Y_{bus} is quite simple to find.

For large systems this approach becomes unwieldly because of the need of inverting a very large matrix. It then becomes much more computer efficient to use direct assembly methods for Z_{bus}.

In order to understand the formation of n-bus system Z-matrix consider again the three-bus example given above. We have redrawn the Thevenin network in Fig. 5 as a "three-port" network shown in Fig. . For this network the Z_{bus} formulation read in detailed from

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3$$
$$V_3 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3$$

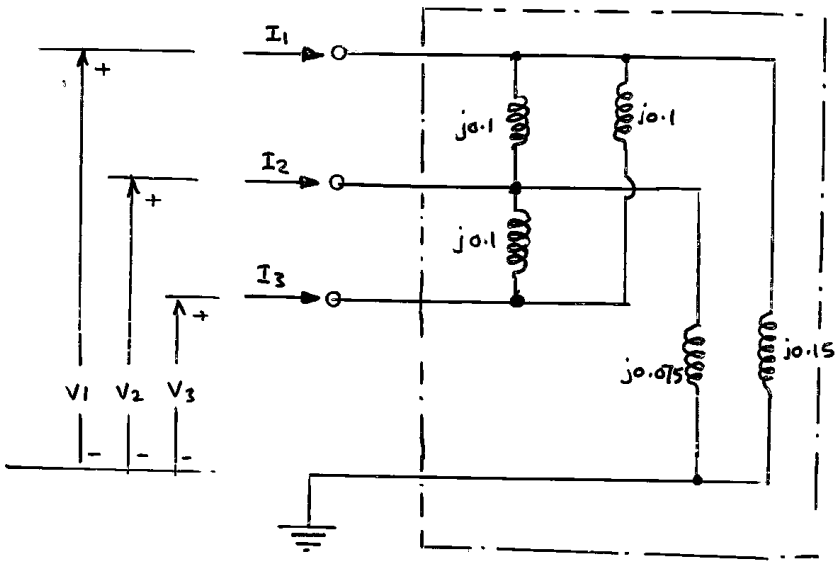


Fig. 5. Three-bus system in Fig. 4 depicated as an "n-port network"

V_1, V_2 and V_3 are the voltages between the ports and the ground reference. I_1, I_2 and I_3 are the injected port currents.

From Fig. 5 we now easily interpret the meaning of the nine elements of the Z_{bus} matrix. For example, if we inject 1A into port 1 but leave port 2 and 3 open, the above three equations are simplified to

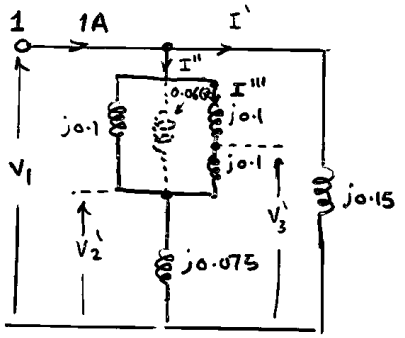
$$V_1 = Z_{11}(1) + Z_{12}(0) + Z_{13}(0) = Z_{11}$$
$$V_2 = Z_{21}(1) + Z_{22}(0) + Z_{23}(0) = Z_{21}$$
$$V_3 = Z_{31}(1) + Z_{32}(0) + Z_{33}(0) = Z_{31}$$

Thus, by computing (or measuring) the three terminal voltages we obtain the first column in the Z_{bus} matrix. For example, the voltage between port 1 and ground (with port 2 and 3 open) equals

$$V_1 = j(1)\ \frac{(0.0667+0.075)\times 0.15}{0.0667+0.075+0.15} = j0.073 \text{ p.u.}$$

which is equall to Z_{11} in the Z-matrix in the previsno example.

$\Longrightarrow Z_{11}$



To find Z_{12} we have to compute V_2' resulting from injecting 1 A at bus No.1 :

$$I'' = (1)\ \frac{j0.15}{(j0.0667+j0.075)+j0.15} = 0.514$$ By Current Division rule

$$\therefore |V_2'| = I'' \times j0.075 = j0.0386$$

which is $Z_{12} = Z_{21}$ in the Z-matrix.

Similarly we can find V_3' :

$$V_3' = V_2' + I''' \times j0.1$$
$$I''' = I'' \frac{j0.1}{j0.1+j0.1+j0.1} = 0.514\ \frac{0.1}{0.3} = 0.1714$$

$$\therefore V_3 = j0.0386 + 0.1714 \times j0.1 = j0.0558.$$

$$\therefore Z_{13} = j0.0558. = Z_{31}$$